More Challenging Quadratic Factoring

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WHAT'S COVERED

In this lesson, you will learn how to factor a quadratic expression in the form ax^2+bx+c. Specifically, this lesson will cover:

1. Review of Basic Quadratic Factoring

Factoring a quadratic is the opposite process of FOILing: we are taking a quadratic expression and rewriting it as two binomials being multiplied together. With basic factoring in the form x^2+bx+c, we are seeking to identify two integers, *p* and *q*, such that:

- The product of *p* and *q* is the constant term of the quadratic
- The sum of *p* and *q* is the coefficient of the x-term in the quadratic

> EXAMPLE The quadratic expression x^2-5x+6 factors to $(x-2)(x-3)$, because $(-2)\cdot(-3)=6$, which is the constant term, and $-2+(-3)=-5$, which is the coefficient of the x-term.

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2. Factoring when $a \neq 1$

Things get more complicated when the leading coefficient of the quadratic is other than 1, like $2x^2+9x-5$. This is because at least one of the binomials in factored form must have a coefficient of *x* other than 1, like $(2x-1)$. If we start with FOIL, we can see this pattern:

$$\begin{aligned}&(ax+p)(bx+q)\\&=abx^2+aqx+bpx+pq\\&=abx^2+(aq+bp)x+pq\end{aligned}$$

When factoring these kinds of quadratics, we still start with the product of *p* and *q*, *pq*. We break down the constant term into two integers that will multiply to equal that constant term. However, when we add the numbers together to match up with the x-term coefficient, we must first multiply *q* by a (this is seen in the aqx term above) and also *p* by *b* (this is seen in the *bpx* term above).
Let's use this pattern to factor the following quadratic:

↪ EXAMPLE Factor $3x^2 - 13x - 10$.

We know our solution will be in the form $(ax+p)(bx+q)$, so we can fill out part of it right now. Since 3 is a prime number, our only option for the factoring will be:

$$(3x+p)(x+q)$$

This tells us that we will actually only multiply *q* by 3, but don't need to do anything with*p* because it is multiplied by the coefficient of *bx*, which is technically just 1. Now let's break down the constant term, -10, into two integers to find *p* and *q*.

p	*q*
1	-10
2	-5
-1	10
-2	5

These are our candidates for what *p* and *q* can be. To determine which pair of integers we will use, we must first multiply *q* by 3 and then add to*p*:

p	+	**3*q***	=	*p*+**3*q***
1	+	-30	=	-29
2	+	-15	=	-13
-1	+	30	=	29
-2	+	15	=	13

Since we are looking for a sum of -13, we could have stopped after the second pair and identified *p* as 2 and *q* as -5 to use when factoring. This means that we can factor the quadratic as:

$$3x^2 - 13x - 10 = (3x+2)(x-5)$$

HINT

You can always test our your answer by using FOIL:

$(3x+2)(x-5)$	FOIL the factored form by multiplying the first terms, $3x \cdot x = 3x^2$
$3x^2$	Multiply the outside terms, $3x \cdot -5 = -15x$
$3x^2 - 15x$	Multiply the inside terms: $x \cdot 2 = 2x$
$3x^2 - 15x + 2x$	Multiply the last terms: $2 \cdot -5 = -10$

$3x^2 - 15x + 2x - 10$ Combine like terms

$3x^2 - 13x - 10$ Our solution

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3. The Box Method

Another method for factoring quadratics with a leading coefficient other than 1 is to use the box method. In the box method, we'll be able to find the factored form, $(ax+p)$ and $(bx+q)$, on the outside with the expanded form on the inside.

Here is how to display the factors for the quadratic expression $2x^2 - 5x + -12$ using the box method.

	$2x$	+3
x	$2x^2$	$3x$
-4	$-8x$	-12

Notice the inside of the box is an expanded form of $2x^2 - 5x + -12$. The numbers $2x^2$, $3x$, $-8x$, and -12 can be combined to get the original expression of $2x^2 - 5x + -12$.

Also notice how we can multiply the factors at the top to get the answers inside the box, for example, $2x \cdot x = 2x^2$, $2x \cdot -4 = -8$, $3 \cdot x = 3x$, and $3 \cdot -4 = -12$. With the box method, we'll fill out the inside of the box first and work our way backward to find the outside factors.

↱ EXAMPLE Factor $3x^2 - 13x - 10$.

To start, write the original x-squared term and the constant term in the corners of a 2x2 grid:

$3x^2$	
	-10

Next, we multiply the leading coefficient of x-squared by the constant term and identify two integers that multiply to this value. In this case, we multiply 3 times -10, so we are going to list two integers that multiply together to equal -30.

p	q

1	-30
2	-15
3	-10
5	-6
-1	30
-2	15
-3	10
-5	6

Once again, we want the sum of these two integers to match the x-term coefficient, which is -13. From our list, we will use 2 and -15. However, these two values are not used in the same way as before. We already know that $(x+2)(x-15)$ does not equal $3x^2-13x-10$. Instead, we use these two numbers as coefficients to x terms to be included in the 2x2 box, along with the x-squared term and the constant term.

$3x^2$	$2x$
$-15x$	-10

It does not matter which squares these terms go into. Either way, you'll get the same answer. Also, note that the sum of all of the terms in the square equals our quadratic. Adding the four boxes, we see that $3x^2+2x-15x-10$ is equal to $3x^2-13x-10$, our original quadratic. If the sum doesn't add up, you've made an error somewhere.

So what do we do to this grid now? We look at the rows and columns, factor out the greatest common factor in each row and column, and write it outside of the grid. What's left outside of the grid are the two factors to write the quadratic in factored form.

Factor $3x$ out of $3x^2$ and -15x.

	$3x$	
	$3x^2$	$2x$
	$-15x$	-10

Factor 2 out of $2x$ and -10:

	$3x$	$+2$
	$3x^2$	$2x$
	$-15x$	-10

Factor x out of $3x^2$ and $2x$:

	$3x$	$+2$
x	$3x^2$	$2x$
	$-15x$	-10

Factor -5 out of $-15x$ and -10:

	$3x$	$+2$
x	$-3x^2$	$2x$
-5	$-15x$	-10

What we factored outside of the box forms the binomials used in factored form:

	$3x$	$+2$
x	$-3x^2$	$2x$
-5	$-15x$	-10

We can write this as:

$$3x^2 - 13x - 10 = (3x+2)(x-5)$$

Let's double-check our answer by using FOIL:

$$\begin{aligned}&(3x+2)(x-5)\\&3x \cdot x + 3x \cdot -5 + 2 \cdot x + 2 \cdot -5\\&3x^2 - 15x + 2x - 10\\&3x^2 - 13x - 10\end{aligned}$$

SUMMARY

Factoring is the process of writing an equation from expanded form to factored form. A **review of basic quadratic factoring** (when the x-squared coefficient is equal to 1), involves finding two integers whose sum is the coefficient of the x term and whose product is the constant term. **Factoring when $a \neq 1$** involves multiplying the leading coefficient of x-squared, or a, by the constant term, and identify two integers that multiply to this value. This is also known as **the box method**.

