Multiplying and Dividing Functions

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to multiply or divide two functions. Specifically, this lesson will cover:

1. Multiplying Two Functions

Oftentimes when working with functions, we will be asked to determine the result of multiplying two functions. When this occurs, there are two ways to approach the problem.

- **Method 1: Different Values of** ***x*** If the two functions are in terms of the same variable BUT being evaluated for different values, we can evaluate each function for the value we are given and then multiply.
- **Method 2: Same Values of** ***x*** If the two functions are in terms of the same variable and are being evaluated for the same value, we can multiply both functions together and substitute the value we are given to find a solution. Let's look at both methods.

1a. Method 1: Different Values of *x*

When given two functions evaluated at different values, evaluate each function separately then simply multiply the results together.

EXAMPLE Find the value of $f(3)g(2)$ when $f(x)=x^2-2$ and $g(x)=x+7$.

To solve this problem, first evaluate the functions $f(x)$ when x equals 3 and $g(x)$ when x equals 2, separately.

$f(x)=x^2-2$	Find $f(3)$ by substituting x with 3
$f(3)=(3)^2-2$	Evaluate 3^2
$f(3)=9-2$	Subtract 2 from 9
$f(3)=7$	Our solution for $f(3)$
$g(x)=x+7$	Find $g(2)$ by substituting x with 2
$g(2)=2+7$	Add 2 and 7
$g(2)=9$	Our solution for $g(2)$

$f(3)g(2) = 7 \cdot 9$	Find $f(3)g(2)$ by multiplying 7 and 9
$f(3)g(2) = 63$	Our solution

1b. Method 2: Same Values of *x*

When multiplying two functions given in terms of the same variable and evaluated at the same value we can multiply both equations first and then evaluate for the value we are given. Often times the notation used for this is $f(a)g(a) = (f \cdot g)(a)$.

EXAMPLE Find the value of $f(3)g(3)$ using the same functions given in Method 1.

$$f(x) = x^2 - 2$$
$$g(x) = x + 7$$

Because each function is dependent on *x* and we are evaluating it for them for the same value, 3, we can multiply both functions first and then evaluate them for the given value. Let's look at how to do this.

$f(x)g(x)$	Substitute $f(x)$ and $g(x)$ with their equivalent expressions
$f(x)g(x) = (x^2 - 2)(x + 7)$	FOIL
$f(x)g(x) = x^3 + 7x^2 - 2x - 14$	Plug in 3 for *x*
$f(3)g(3) = (3)^3 + 7(3)^2 - 2(3) - 14$	Evaluate
$f(3)g(3) = 27 + 63 - 6 - 14$	Simplify
$f(3)g(3) = 70$	Our solution

---

2. Dividing Two Functions

When we divide two functions we go through a process very similar to that used when multiplying two functions; the only difference is that we have to perform division instead of multiplication.

2a. Method 1: Different Values of *x*

When there is division between two functions and the values are different, you can first evaluate each function for the given value and then perform the division.

EXAMPLE Find the value of $\frac{f(2)}{g(3)}$ when $f(x) = x^2 - 1$ and $g(x) = 3x$.

Since the *x* value is different in each function, evaluate $f(2)$ and $g(3)$ separately, then divide the results.

$f(2) \div g(3)$	Evaluate $f(2)$
$f(2) = (2)^2 - 1 = 3$	Evaluate $g(3)$
$g(3) = 3(3) = 9$	Divide $f(2)$ by $g(3)$

$$\frac{f(2)}{g(3)} = \frac{3}{9} \quad \text{Simplify}$$

$$\frac{f(2)}{g(3)} = \frac{1}{3} \quad \text{Our solution}$$

2b. Method 2: Same Values of *x*

Similar to Method 2 discussed above for multiplying two functions, if the two functions are dependent on the same variable and evaluated for the same value, we can also perform division between two functions first and then evaluate for a given value.

↱ EXAMPLE Find the value of $\frac{f(2)}{g(2)}$ using the same functions given above.

$f(x) = x^2 - 1$ $g(x) = 3x$	Substitute $f(x)$ and $g(x)$ with their equivalent expressions
$\frac{f(x)}{g(x)} = \frac{x^2 - 1}{3x}$	Substitute 2 in for x
$\frac{f(2)}{g(2)} = \frac{2^2 - 1}{3(2)}$	Evaluate numerator and denominator
$\frac{f(2)}{g(2)} = \frac{3}{6}$	Simplify
$\frac{f(2)}{g(3)} = \frac{1}{2}$	Our solution

As with multiplying two functions you can use either method shown above the first method works quite well for simple functions but the second method can be much more useful when working with more complex functions. Unlike with multiplication, you have to be careful that you do not divide by 0 when dividing two functions. If we have the generic form $f(x)/g(x)$, when $g(x)$ equals 0, the function will be undefined.

BIG IDEA

When dividing two functions, you can evaluate each function, and then divide. If both functions are defined by the same variable and evaluated at the same value, we can divide both functions first and then evaluate for the given value. In the second case, the notation used is $f(a)/g(a) = (f/g)(a)$.

HINT

We must be sure that the function in the denominator does not equal zero. If this does happen, then our function would be "undefined" or we would simply have "no solution." Note that the function in the denominator may return an undefined solution for only a certain domain. In our previous example, $f(x)/g(x)$ would be undefined at $x = 3$, because $g(3) = 0$, however the function would have a value for other values of *x*.

SUMMARY

When **multiplying or dividing functions**, evaluate each function separately and combine the values for each function. For a given value a in the domain of $f(x)$ and $g(x)$, $f(a)g(a)$ equals $(f \cdot g)(a)$. Similarly,

$f(a)/g(a)$ equals $(f/g)(a)$.