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Multiplying Polynomials

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WHAT'S COVERED

In this lesson, you will learn how to multiply three binomials. Specifically, this lesson will cover:

1. Distribution

When multiplying two expressions, we can distribute all factors into each term of the other expression. With simple algebraic expression, this typically involves a coefficient and a single variable, as in the example below:

EXAMPLE Distribute $-4(x+6)$.

$-4(x+6)$	Distribute -4 into each term in the parentheses
$-4 \cdot x+(-4) \cdot 6$	Evaluate the multiplication
$-4x-24$	Our solution

We can also use the distribution property with examples involving other variables or exponents:

EXAMPLE Multiply $5x(3y+x^2)$.

$5x(3y+x^2)$	Distribute $5x$ into each term in the parentheses
$5x \cdot 3y+5x \cdot x^2$	Evaluate the multiplication
$15xy+5x^3$	Rewrite in descending order of degree
$5x^3+15xy$	Our solution

2. FOIL

In the previous examples, we multiplied a monomial (single-term expression) by a binomial (two-term expression). When multiplying two binomials together, we use a special case of the distributive rule, commonly referred to as FOIL. As we have learned earlier in the course, FOIL stands for First, Outside, Inside, Last, and describes how to distribute all terms in binomial multiplication.

↪ EXAMPLE Multiply $(2x-3)(x+4)$.

$(2x-3)(x+4)$	Multiply the first terms to start the calculation, $2x \cdot x = 2x^2$
$2x^2$	Multiply the outside terms, $2x \cdot 4 = 8$, and add to the calculation
$2x^2 + 8x$	Multiply the inside terms, $-3 \cdot x = -3x$, and add to the calculation
$2x^2 + 8x - 3x$	Multiply the last terms, $-3 \cdot 4 = -12$, and add to the calculation
$2x^2 + 8x - 3x - 12$	Combine like terms, $8x$ and $-3x$
$2x^2 + 5x - 12$	Our solution

HINT

As you are multiplying with FOIL, you can also just write this as one, long expression and then evaluate:

$$(2x-3)(x+4)$$
$$\underbrace{(2x)(x)}_{first} + \underbrace{(2x)(4)}_{outside} + \underbrace{(-3)(x)}_{inside} + \underbrace{(-3)(4)}_{last}$$
$$2x^2 + 8x - 3x - 12$$
$$2x^2 - 5x - 12$$

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3. Multiplying Three Binomials

How can the distributive and FOIL processes be modified to multiply three binomials? One strategy is to perform the steps in FOIL to two of the binomials and then distribute the third binomial into the product created by FOIL.

↪ EXAMPLE Multiply $(2x-1)(x+1)(x-3)$.

$(2x-1)(x+1)(x-3)$	Choose two binomials to FOIL, for instance, $(x+1)$ and $(x-3)$
$(2x-1)(x+1)(x-3)$	FOIL the two binomials
$(2x-1)\left(\underbrace{(x \cdot x)}_{first} + \underbrace{(x \cdot -3)}_{outside} + \underbrace{(1 \cdot x)}_{inside} + \underbrace{(1 \cdot -3)}_{last}\right)$	Evaluate the multiplication
$(2x-1)(x^2 - 3x + x - 3)$	Combine like terms
$(2x-1)(x^2 - 2x - 3)$	The solution to the two binomials

So far, all that we have done is used FOIL to multiply two of the three binomials. In order to multiply the third binomial, we will distribute each term of $2x-1$ into every term in $x^2 - 2x - 3$. In order to keep things organized, it is helpful to distribute separately, and then add the two new polynomials:

First, multiply $2x$ by $x^2 - 2x - 3$. Then, multiply -1 by $x^2 - 2x - 3$:

$$2x(x^2-2x-3)=2x^3-4x^2-6x$$
$$-1(x^2-2x-3)=-x^2+2x+3$$

Our final step is to add these two polynomials. Remember to use coefficients of 0 to keep the vertical alignment between like terms.

$$\begin{array}{r} (2x^3-4x^2-6x+0) \\ +\underline{(0x^3-1x^2+2x+3)} \\ 2x^3-5x^2-4x+3 \end{array}$$

SUMMARY

When multiplying polynomials, you need to use **distribution** and multiply each term in the first polynomial by each term in the second polynomial. This is also referred to as **FOIL**. When **multiplying three binomials**, you should first multiply or FOIL two factors together and combine any like terms to simplify the polynomial. Then multiply the third factor. Finally, you should write the polynomial in standard form, ordering the terms by the degree from highest to lowest.