Negative Exponents

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WHAT'S COVERED

In this lesson, you will learn how to simplify an expression with negative exponents. Specifically, this lesson will cover:

1. Zero Property

There are a few special exponent properties that deal with exponents that are not positive. The first is considered in the following example, which is worded out 2 different ways:

EXAMPLE Use the quotient property and solve.

$\frac{a^3}{a^3}$	Use the quotient rule to subtract exponents
a^0	Our Solution

But now we consider the same problem in a second way:

EXAMPLE Rewrite the exponents as repeated multiplication and solve.

$\frac{a^3}{a^3}$	Rewrite exponents as repeated multiplication
$\frac{aaa}{aaa}$	Reduce out all the $a's$

1 Our Solution

When we combine these two results, we get $a^0 = 1$. This final result is an important property known as the **zero property of exponents**:

FORMULA TO KNOW

Zero Property of Exponents

$a^0 = 1$

Any number or expression raised to the zero power will always be 1.

EXAMPLE

$(3x^2)^0$	Zero power rule
1	Our Solution

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2. Negative Exponents

2a. Rule #1

Another property we will consider here deals with negative exponents. Again we will solve the following example in two ways.

EXAMPLE Use the quotient property and solve.

$\frac{a^3}{a^5}$	Using the quotient rule, subtract exponents
a^{-2}	Our Solution

But now we consider the same problem in a second way:

EXAMPLE Rewrite the exponents as repeated multiplication and solve.

$\frac{a^3}{a^5}$	Rewrite exponents as repeated multiplication
$\frac{aaa}{aaaaa}$	Reduce three $a's$ out of top and bottom
$\frac{1}{aa}$	Simplify to exponents
$\frac{1}{a^2}$	Our Solution

When we combine these two results, we get $a^{-2} = \frac{1}{a^2}$. This example illustrates an important property of exponents. Negative exponents yield the reciprocal of the base. Once we take the reciprocal the exponent is now positive. Also, it is important to note a negative exponent does not mean the expression is negative, only that we need the reciprocal of the base. This gives us Rule #1 of the **properties of negative exponents**.

FORMULA TO KNOW

Properties of Negative Exponents

Rule #1: $a^{-n} = \frac{1}{n}$

2b. Rule #2

Negative exponents can be combined in several different ways. As a general rule if we think of our expression as a fraction, negative exponents in the numerator must be moved to the denominator, likewise, negative exponents in the denominator need to be moved to the numerator. When the base with the exponent moves, the exponent is now positive.

EXAMPLE

$\frac{a^3b^{-2}c}{2d^{-1}e^{-4}f^2}$	Negative exponents on *b*, *d*, and *e* need to flip
$\frac{a^3cde^4}{2b^2f^2}$	Our Solution

This gives us Rule #2 of the **properties of negative exponents**.

FORMULA TO KNOW

Properties of Negative Exponents

Rule #2: $\frac{1}{a^{-n}} = a^n$

As we simplified our fraction we took special care to move the bases that had a negative exponent, but the expression itself did not become negative because of those exponents. Also, it is important to remember that exponents only affect what they are attached to. The 2 in the denominator of the above example does not have an exponent on it, so it does not move with the *d*.

2c. Rule #3

What if you had an expression with a fraction and there was a negative exponent applied to the whole fraction?

EXAMPLE

$\left(\frac{a}{b}\right)^{-2}$	Invert the numerator and denominator and change the negative exponent to a positive exponent.
$\frac{b^2}{a^2}$	Our Solution

The result is simply the reciprocal of the fraction, where each term in the numerator and denominator is raised to a positive power *n*. This gives us Rule #3 of the **properties of negative exponents**.

FORMULA TO KNOW

Properties of Negative Exponents

Rule #3: $\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$

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3. Simplifying With Negative Exponents

Simplifying with negative exponents is much the same as simplifying with positive exponents. It is advised to keep the negative exponents until the end of the problem and then move them around to their correct location (numerator or denominator). As we do this, it is important to be very careful of rules for adding, subtracting, and multiplying with negatives.

EXAMPLE

$\frac{x^{-5}x^7}{x^{-4}}$	Simplify numerator with product rule, adding exponents
$\frac{x^2}{x^{-4}}$	Use Quotient rule to subtract exponents, be careful with the negatives!
x^6	Our Solution

EXAMPLE

$\frac{4x^{-5}y^{-3}\cdot 3x^3y^{-2}}{6x^{-5}y^3}$	Simplify numerator with product rule, adding exponents
$\frac{12x^{-2}y^{-5}}{6x^{-5}y^3}$	Quotient rule to subtract exponents, be careful with negatives! $(-2)-(-5)=(-2)+5=3$ $(-5)-3=(-5)+(-3)=-8$
$2x^3y^{-8}$	Negative exponent needs to move down to denominator
$\frac{2x^3}{y^8}$	Our Solution

EXAMPLE

$\frac{(3ab^3)^{-2}ab^{-3}}{2a^{-4}b^0}$	In numerator, use power rule with -2, multiplying exponents. In denominator, $b^0=1$
$\frac{3^{-2}a^{-2}b^{-6}ab^{-3}}{2a^{-4}}$	In numerator, use product rule to add exponents

$\dfrac{3^{-2}a^{-1}b^{-9}}{2a^{-4}}$ Use quotient rule to subtract exponents, be careful with negatives $(-1)-(-4)=(-1)+4=3$

$\dfrac{3^{-2}a^{3}b^{-9}}{2}$ Move 3 and b to denominator because of negative exponents

$\dfrac{a^{3}}{3^{2}2b^{9}}$ Evaluate $3^{2}2$

$\dfrac{a^{3}}{18b^{9}}$ Our solution

HINT

In the previous example it is important to point out that when we simplified 3^{-2} we moved the three to the denominator and the exponent became positive. We did not make the number negative! Negative exponents never make the bases negative, they simply mean we have to take the reciprocal of the base.

SUMMARY

You can rewrite any **negative exponent** as positive using one of these two properties. Any base, *b*, to a negative exponent, *-n* can be written as 1 over the same base, *b*, to a positive exponent, *n*. The exponent goes from negative to positive. We now have our base and exponent in the denominator of the fraction. It's like we have flipped the fraction.

Similarly, if you have a fraction, 1 over base, *b*, to a negative exponent, *-n*, we can write it as the same base *b* to the positive exponent *n*. Again, our exponent goes from negative to positive. Instead of the base and exponent being in the denominator of the fraction, we have it written by itself.

FORMULAS TO KNOW

Properties of Negative Exponents

Rule #1: $a^{-n}=\dfrac{1}{a^{n}}$

Rule #2: $\dfrac{1}{a^{-n}}=a^{n}$

Rule #3: $\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$

Zero Property of Exponents

$a^0 = 1$