One to One Functions

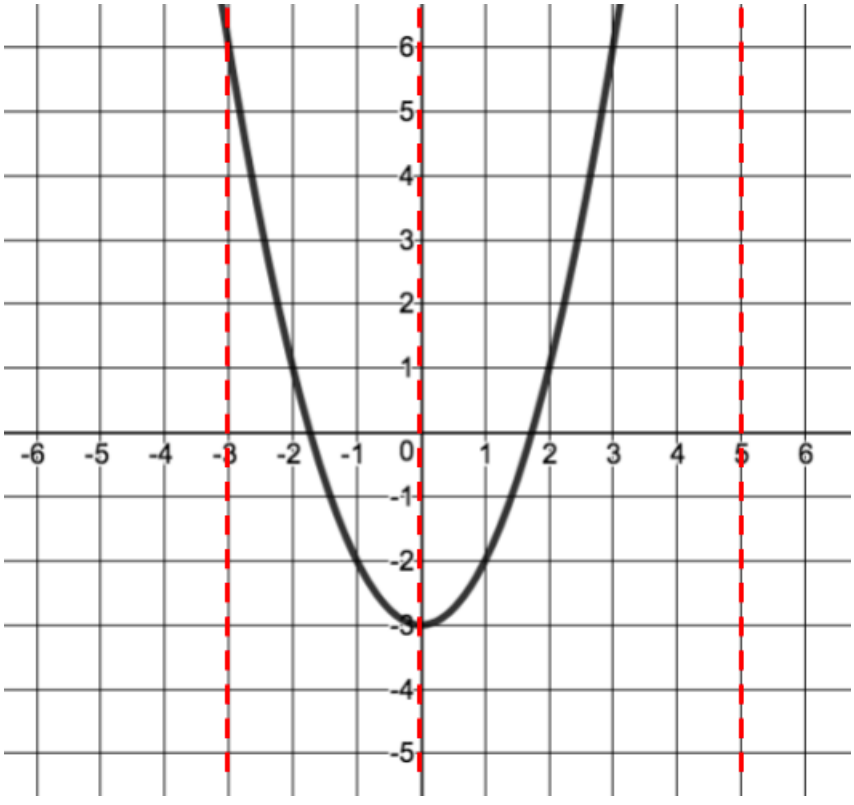
by Sophia

WHAT'S COVERED

In this lesson, you will learn how to identify the graph of a one-to-one function. Specifically, this lesson will cover:

1. Function Review

A defining characteristic of a function is that for every element in the domain/input, there is exactly one corresponding element in the range/output. If we look at this graphically, we we will see that the graph of a function passes the Vertical Line Test, where a vertical line is drawn, and if the graph does not touch the line in more than one place we consider this a function.



BIG IDEA

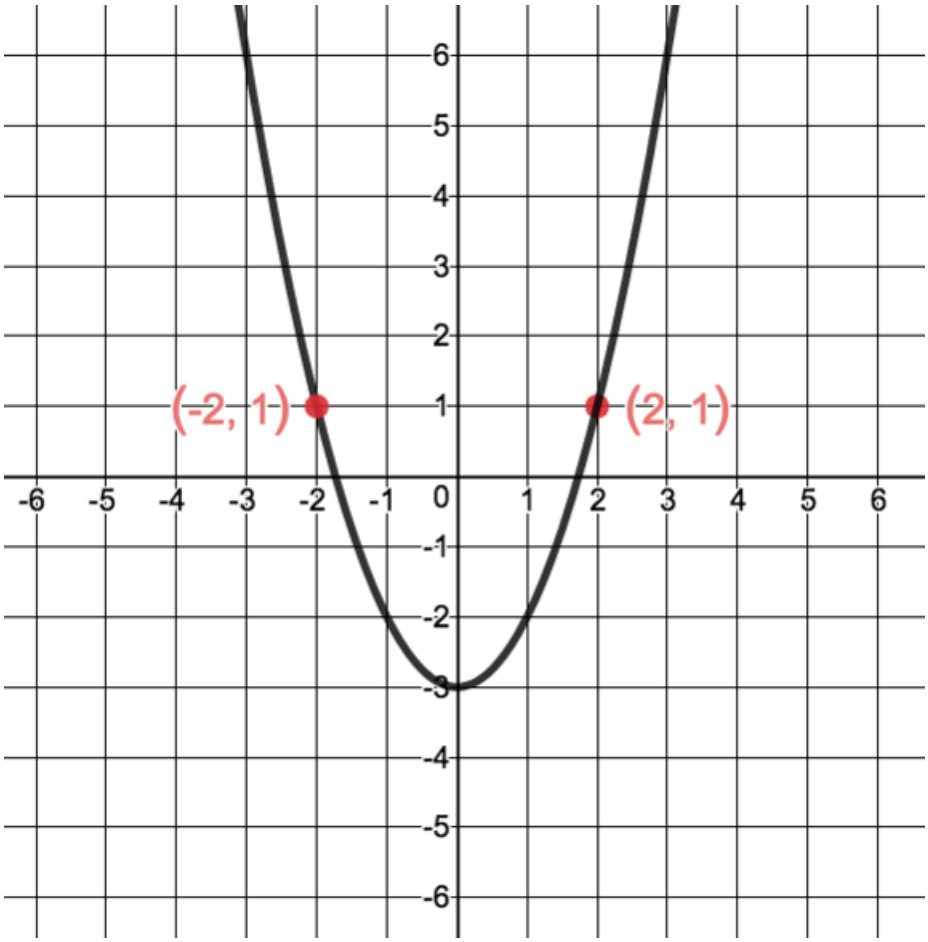
Notice how the graph does not touch each vertical line more than once. When this occurs, we say that the graph represents a function. What this means that is that each input, or x-value, of the function

corresponds to exactly one output, or y-value which is the same as an $f(x)$ value.

2. Introduction to One-to-One Functions

One-to-One functions are special types of functions where every value in the domain of the function corresponds to only one value in the range and each value in the range corresponds to one one value in the domain.

Notice that in the graph shown above, the function is NOT a one-to-one function. This is because there is at least one instance where two or more x-values result in the same y-value, for example (-2, 1) and (2, 1).



3. Determining if a Function is One-to-One

In order to determine if a function is one-to-one we can use two methods:

- Graphically, where we perform both a Vertical Line Test and a Horizontal Line Test.
- Algebraically, where we use two values a and b to find $f(a) = f(b)$ and manipulate the problem to show that $a = b$.

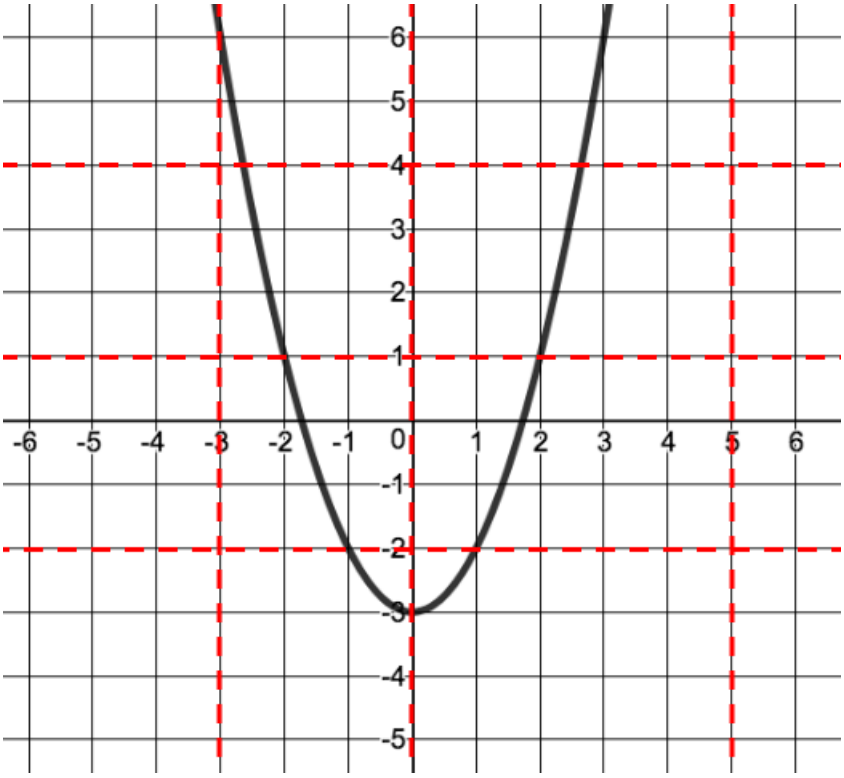
3a. Graphically

Given a graph of a function, we can simply draw vertical and horizontal lines on the graph to help determine if the graph represents a one-to-one function. If the graph only touches each line once then we may be safe in

saying that the graph represents a one-to-one function.

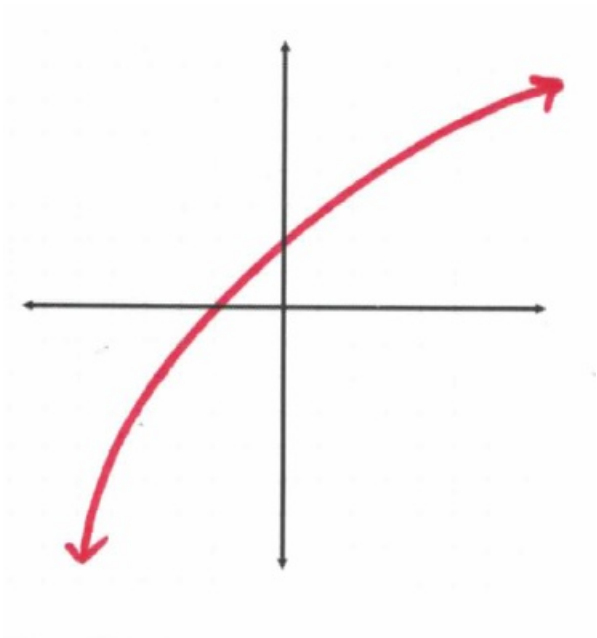
In a Horizontal Line Test, horizontal lines are drawn on the coordinate plane and we try to determine how many times the graph of a function touches each horizontal line. If the graph only touches each horizontal line once we say that the graph passes the Horizontal Line Test.

↱ EXAMPLE Determine if the following graph is one-to-one.



Although it passes the Vertical Line Test, this graph does not pass the Horizontal Line Test, so this graph does not represent a one-to-one function.

↱ EXAMPLE Determine if the following graph is one-to-one.



This function is one-to-one because it passes both the vertical and horizontal line tests.

BIG IDEA

If a graph passes both the Vertical and Horizontal Line Tests, then the graph represents a one-to-one function.

3b. Algebraically

Sometimes a graph of a function may be too large to draw on a coordinate plane so it can be difficult to determine if the graph represents a one-to-one function. In such cases, is it better to determine if a function is one-to-one algebraically. To determine if a function is one-to-one algebraically we do the following:

STEP BY STEP

1. Given a function $f(x)$, use two values a and b to find $f(a)$ and $f(b)$.
2. Assume $f(a)=f(b)$.
3. Manipulate the problem to show that $a=b$.

If we can prove that $a=b$, then we are dealing with a one-to-one function.

↷ EXAMPLE Determine if the function $f(x)=x^3+4$ is one-to-one.

First, we use two values a and b to find $f(a)$ and $f(b)$.

$$f(a)=a^3+4$$
$$f(b)=b^3+4$$

Next, we set them equal to each other and then manipulate the problem to see if $a=b$.

$f(a)=f(b)$	Substitute expressions for $f(a)$ and $f(b)$
$a^3+4=b^3+4$	Subtract 4 from both sides
$a^3=b^3$	Take cube-root of both sides
$\sqrt[3]{a^3}=\sqrt[3]{b^3}$	Simplify
$a=b$	Our solution

Since we were able to get $a=b$, the function $f(x)=x^3+4$ is one-to-one.

SUMMARY

Reviewing functions, recall that a defining characteristic of a function is that for every element in the domain/input, there is exactly one corresponding element in the range/output. **One-to-One functions** not only pass the Vertical Line Test, but they also must pass the Horizontal Line Test. We can **determine if a function is one-to-one graphically** by looking at using these tests, Passing the Vertical Line Test means that there's exactly one value for *y* at any given *x* value and passing the Horizontal

Line Test means that there's exactly one value for x at any given y value. To **determine if a function is one-to-one algebraically**, we need to test to see if a equal b if $f(a)$ equal $f(b)$.

Source: ADAPTED FROM "BEGINNING AND INTERMEDIATE ALGEBRA" BY TYLER WALLACE, AN OPEN SOURCE TEXTBOOK AVAILABLE AT **www.wallace.ccfaculty.org/book/book.html**. License: Creative Commons Attribution 3.0 Unported License