Sophia

Operations as Grouping Symbols

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to evaluate an expression with absolute value using order of operations. Specifically, this lesson will cover:

1. Operations as Grouping Symbols

The order of operations outlines an order we should follow to evaluate expressions with multiple operations. We often use the acronym **PEMDAS** to remember the order of operations:

It is important to remember that the "parentheses" part of the order of operations includes grouping symbols, such as braces or brackets. There are several types of operations that act as grouping symbols. This means that they are operations, but need to be considered first in the order of operations, because they also enclose other expressions in parentheses, even though they might not be visible.

In this lesson, we will see three operations that act as grouping symbols: fraction bars, absolute value bars, and the radical symbol.

TERM TO KNOW

PEMDAS

An acronym to remember the order of operations: parentheses, exponents, multiplication and division (from left to right), addition and subtraction (from left to right).

1a. Fraction Bars

If we have a fraction, the entire numerator and the entire denominator must be evaluated before we reduce the fraction. In these cases, we can simplify both the numerator and denominator at the same time.

EXAMPLE

$$\frac{2^4-(-8)\cdot 3}{15\div 5-1}$$ Exponent in the numerator, divide in denominator

$$\frac{16-(-8)\cdot 3}{3-1}$$ Multiply in the numerator, subtract in the denominator

$$\frac{16-(-24)}{2}$$ Add the opposite to simplify numerator, denominator is done

$$\frac{40}{2}$$ Reduce, divide

20 Our Solution

BIG IDEA

A fraction bar groups numbers and operations in a numerator and denominator. Although a fraction bar represents division, it creates implied parentheses around everything on top and on the bottom. We must fully evaluate both the numerator and denominator of a fraction, and divide at the very end.

1b. Absolute Value Bars

Another type of grouping symbol that also has an operation with it is absolute value. When we have an absolute value, we will evaluate everything inside the absolute value bars, just as if it were a normal parentheses. Then once the inside is completed, we will take the absolute value, or distance from zero, to make sure the number is non-negative.

EXAMPLE

$1+3\left\|-(4^2)-(-8)\right\|+2\left\|3+\underline{(-5)^2}\right\|$	Evaluate absolute first, exponents
$1+3\left\|\underline{-16-(-8)}\right\|+2\left\|\underline{3+25}\right\|$	Add inside absolute values
$1+3\left\|\underline{-8}\right\|+2\left\|\underline{28}\right\|$	Evaluate absolute values
$1+\underline{3(8)}+2(28)$	Multiply left to right
$1+24+\underline{2(28)}$	Finish multiplying
$\underline{1+24}+56$	Add left to right
$\underline{25+56}$	Add
81	Our Solution

BIG IDEA

Absolute value bars also act as a grouping symbol, and come first in the order of operations. If we see an expression inside absolute value bars, we must completely evaluate that expression, then take its absolute value, before moving on to the other operations according to the order of operations.

HINT

The above example also illustrates an important point about exponents. Exponents are considered to be only on the number they are attached to. This means when we see -4^2, only the 4 is squared, giving us $-(4^2)$ or -16. But when the negative is in parentheses, such as $(-5)^2$, the negative is part of the number and is also squared, giving us a positive solution, 25.

1c. Radical Symbols

Expressions can be underneath radical symbols, just like they can be in numerators, denominators, or within absolute value bars. When an expression is underneath a radical, this groups the expression together, and acts as a grouping symbol. This means we must fully evaluate the expression underneath the radical sign, before moving on to other operations in the order of operations.

EXAMPLE

$\sqrt{4 \cdot 5+5}+7$	The radical sign groups the expression $4 \cdot 5+5$
$\sqrt{20+5}+7$	Multiply 4 by 5
$\sqrt{25}+7$	Add 5
$5+7$	Evaluate the square root
12	Our Solution

SUMMARY

Grouping symbols fall under parentheses in PEMDAS, which means that when you're evaluating with the order of operations, you're going to start with grouping symbols first. Grouping symbols include **fraction bars**, **absolute value signs**, and **radicals**. For nested grouping symbols, where you have more than one grouping symbol inside of another one, you need to start by evaluating them from the inside out. Whatever your innermost grouping symbol is, start with that, and then work your way out.

TERMS TO KNOW

PEMDAS

An acronym to remember the order of operations: parentheses, exponents, multiplication and division, addition and subtraction.