Poisson Distribution

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WHAT'S COVERED

This tutorial is going to teach you about the Poisson distribution, which is a special case of the binomial distribution. Our discussion breaks down as follows:

1. Poisson Distribution

Suppose you have a particularly busy intersection. That intersection averages about 4.3 accidents per week. Now, that's not a whole lot of accidents per week for a busy intersection with a lot of cars passing through it. The probability of having an accident at that intersection is fairly small. All other things being equal, what's the probability that the intersection experiences a week with just one accident?

This distribution can be solved using something called the **Poisson distribution**. It's a distribution that works well for rare events--rare meaning the probability of success is very low, but the number of trials is very high. Out of a very large number of trails, you end up with only a few successes.

The Poisson distribution is nice because it doesn't require the use of n and p--all you need is the typical rate of occurrence and the number of events that you are anticipating to occur during this time frame. The following formula demonstrates this relationship:

FORMULA TO KNOW

Poisson Distribution

$$P(X=k)=\frac{\lambda^k e^{-\lambda}}{k!}$$

k *is the given number of event occurrences*

λ *is the average rate of event occurrences*

If the average number of successes in a given time frame is the Greek letter lambda and x is the number of potential successes that you could have, the probability that you get exactly k successes is equal to lambda to the k--the expected number to the number of successes--times the number e to the negative lambda divided by k factorial.

HINT

Recall that the number e is also known as Euler's number. It is an irrational number, meaning it cannot be written as a simple fraction. When using this formula, there should be a button for e on your calculator that you can easily use.

So, k factorial (k followed by the exclamation point) means that you start from k and multiply by k minus 1, k minus 2, all the way down to 1. That means that 4 factorial would be 4 times 3 times 2 times 1.

Going back to our busy intersection example, lambda--the expected rate of occurrence--was 4.3. The number of successes that you wanted this week was 1.

$$k = \text{given number of event occurrences} = 1$$
$$\lambda = \text{average rate of event occurrences} = 4.3$$

$$P(X=1) = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{(4.3)^1 e^{-4.3}}{1!} = 0.058$$

Simplify it out, and you get 0.058, meaning there's about a 6% chance of just one accident.

HINT

If you had chosen zero accidents, which you could have calculated, the 0 factorial is 1. This is something to be aware of: The 0 factorial is 1.

TRY IT

On an average day, 5 iPhone owners under warranty have a hardware failure. What is the likelihood that tomorrow 6 owners experience failure?

$$k = \text{given number of event occurrences} = 6$$
$$\lambda = \text{average rate of event occurrences} = 5$$

$$P(X=6) = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{5^6 e^{-5}}{6!} = 0.146$$

The probability that six iPhone owners will experience a hardware failure is about 15%.

HINT

When evaluating this problem, 6! is the same as 6 • 5 • 4 • 3 • 2 • 1, which equals 720.

TERM TO KNOW

Poisson Distribution

A distribution used to calculate the probability of a given number of events happening in a fixed interval when the events occur independently and the average rate of occurrence is known

SUMMARY

The Poisson distribution gives probabilities from situations that arise from rare events. The number of trials, n, is high, and p, the probability of success, is low. If you know the average rate of occurrence, you can figure out the probability of some exact number of successes.

Good luck!

TERMS TO KNOW

Poisson Distribution

A distribution used for rare events. It can find the probability of exactly a certain number of successes within a given timeframe, assuming that events occur independently.

FORMULAS TO KNOW

Poisson Distribution

$$P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

k *is the given number of event occurrences*

λ *is the average rate of event occurrences*