Sophia

Polynomials Divided by Monomials

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to divide a polynomial by a monomial. Specifically, this lesson will cover:

1. Basic Factoring

If terms in an expression share a common factor, it can be factored out of the expression. It may be helpful to think of this process as being the opposite of distribution.

EXAMPLE Factor $4x+12$.

$4x+12$	Rewrite each term with the common factor of 4
$(4\cdot x)+(4\cdot 3)$	Factor out the 4 in each term
$4(x+3)$	Our solution

EXAMPLE Factor $-6x+42$.

$-6x+42$	Rewrite each term with the common factor of -6
$(-6\cdot x)+(-6\cdot -7)$	Factor out the -6 in each term
$-6(x+(-7))$	Rewrite parentheses as subtraction
$-6(x-7)$	Our solution

As seen in the second example, you can factor out positive or negative factors. However, be sure to check the signs between terms in parentheses to make sure you have correctly factored a common factor. For the second term, 42, since we factored out a -6, we would multiply -6 by -7 to represent the positive 42.

HINT

You can test your factored answer by using distribution to expand the expression.

EXAMPLE Distribute $-6(x-7)$.

$-6(x-7)$	Distribute -6 into each term in the parentheses

$(-6\cdot x)-(-6\cdot 7)$	Evaluate multiplication
$-6x+42$	Evaluate

2. Factoring out an Algebraic Factor

We can use this factoring technique to factor out more than just numbers. If algebraic expressions share variable factors, we can factor them out as well.

→ EXAMPLE Factor $6x^2+2x$.

$6x^2+2x$	Rewrite each term with factors
$(2\cdot 3\cdot x\cdot x)+(2\cdot x)$	Factor out the common factor of $2x$ from each term
$2x(3x+1)$	Factoring out $2x$

HINT

It helps to break down coefficients into prime factors. Doing so in the example above makes it clearer to see how $2x$ is a common factor. Additionally, if an entire term is the factor being factored out, what remains is 1. This is why the binomial in parentheses above is $(3x+1)$.

3. Dividing a Polynomial by a Monomial

Sometimes when dividing a polynomial by a monomial, there are common factors between the monomial term and terms that make up the polynomial. In this case, it is relatively straightforward to divide coefficients and decrease exponents.

→ EXAMPLE Divide $-6x^3+18x^2-3x$ by $3x$.

$\dfrac{-6x^3+18x^2-3x}{3x}$	Rewrite each term with a common factor of $3x$
$\dfrac{3x(-2x^2)+3x(6x)-3x(1)}{3x}$	Rewrite as separate fractions
$\dfrac{3x(-2x^2)}{3x}+\dfrac{3x(6x)}{3x}+\dfrac{3x(1)}{3x}$	Divide each term in the numerator by the denominator
$-2x^2+6x-1$	Our solution

Of course, such examples that divide nicely between all terms are not always the case when dividing polynomials. When we need to divide a term that doesn't share all common factors, we divide what we can, and express the remainder as a fraction.

To show this, let's look at a different example where the first couple of terms divide evenly, but the last term does not. Take note of how we write the division:

EXAMPLE Divide $18x^3-15x^2+9x+6$ by $3x$.

$\dfrac{18x^3-15x^2+9x+6}{3x}$	Rewrite as separate fractions
$\dfrac{18x^3}{3x}-\dfrac{15x^2}{3x}+\dfrac{9x}{3x}+\dfrac{6}{3x}$	Rewrite each expression with common factors
$\dfrac{3x(6x^2)}{3x}-\dfrac{3x(5x)}{3x}+\dfrac{3x(3)}{3x}+\dfrac{3(2)}{3x}$	Divide the numerator by the denominator in each term using common factors
$6x^2-5x+3+\dfrac{2}{x}$	Our solution

We were able to factor out $3x$ from $18x^3$, $15x^2$, and $9x$. However, the only common factor between 6 and $3x$ is the 3, not the x. So this term becomes a fraction, and due to the common factor of 3, we were able to simplify it to 2 divided by x.

SUMMARY

Recall with **basic factoring**, if terms in an expression share a common factor, it can be factored out of the expression. When **factoring out an algebraic factor**, we can use the same technique and look for expressions that share the same variable factors. When **dividing a polynomial by a monomial**, you can write expressions as products of factors that may cancel out when dividing the expression in the numerator of a fraction by the expression in the denominator. Factors that cancel out must be the same in the numerator and in the denominator.