## Properties of Addition and Multiplication

## by Sophia

## WHAT'S COVERED

In this lesson, you will learn how to identify the commutative property, associative property, and distributive property. Specifically, this lesson will cover:

1. Identity Properties
2. Inverse Properties
3. Commutative Property of Addition and Multiplication
4. Associative Properties of Addition and Multiplication
5. Distributive Property

## 1. Identity Properties

The identity property of addition states that when zero is added to any number, the value does not change. In other words, the identity property of addition tells us that adding zero does not change the value of a number. Generally, we can express this as:

## $\Xi$ FORMULA TO KNOW

Identity Property of Addition

$$
a+0=a
$$

```
EXAMPLE
7+0=7
```

A similar property applies to multiplication. What quantity, when multiplied with any number, does not change the value of that number? When any number is multiplied by 1 , the value does not change. The identity property of multiplication states that any number multiplied by 1 does not change in value. Generally, we can express this as:

## $\int$ FORMULA TO KNOW

Identity Property of Multiplication

$$
a \cdot 1=a
$$

## 2. Inverse Properties

The inverse property of addition states that any number and its opposite sum to zero. We can refer to the opposite of a number as its additive inverse. We can write this generally as:

## $』$ FORMULATO KNOW

Inverse Property of Addition

$$
a+(-a)=0
$$

```
\Leftrightarrow EXAMPLE
9+(-9)=0
```

9 and -9 are opposites of each other. This means that their magnitudes are the same (9), but their signs are different; one is a positive number, while the other is a negative number.

## BIG IDEA

The sum of a number and its opposite is zero.
The inverse property of multiplication states that a number and its reciprocal multiply to 1 . In the same way that a number and its opposite are additive inverses, a number and its reciprocal are multiplicative inverses. The reciprocal of a number can be found by creating a fraction, and flipping the numerator and denominator. Here is our general rule:

## $\int$ FORMULA TO KNOW

Inverse Property of Multiplication

$$
a \times \frac{1}{a}=1
$$

$$
\begin{aligned}
& \Leftrightarrow \text { EXAMPLE } \\
& 9 \times \frac{1}{9}=1
\end{aligned}
$$

9 and $\frac{1}{9}$ are multiplicative inverses, or reciprocals, of one another.

```
& BIG IDEA
```

The inverse property of multiplication dictates that the product of a number and its reciprocal is equal to 1 .

## 3. Commutative Property of Addition and Multiplication

Addition and multiplication is commutative. This means that we can add in any order we wish, and we can multiply in any order we wish. It is important to note that we cannot mix addition and multiplication. These are separate properties, but they behave the same with both operations.

The commutative property of addition is expressed with the following formula:

## $\int$ FORMULA TO KNOW

Commutative Property of Addition

$$
a+b=b+a
$$

```
\Leftrightarrow EXAMPLE
2+3=3+2
```

The expression $2+3$ is the same as the expression ${ }^{3+2}$, because addition is commutative. It does not matter in which order you add, the sum will be 5 in either case.
The same property applies to multiplication as well. It does not matter in which order you multiply, because multiplication is commutative. The general rule for the commutative property of multiplication is:

## 』 FORMULA TO KNOW

## Commutative Property of Multiplication

$a b=b a$

```
\Leftrightarrow EXAMPLE
3.4 = 4.3
```

The expression $3 \cdot 4$ is the same as the expression $4 \cdot 3$, because multiplication is commutative. It does not matter in which order you multiply, the product will be 12 in either case.

## - TERM TO KNOW

Commutative Property
A property of addition that allows terms to be added in any order; a property of multiplication that allows factors to be multiplied in any order.

## 4. Associative Properties of Addition and Multiplication

The associative property allows us to group terms for addition and multiplication in any way we wish. As with the commutative properties of addition and multiplication, the associative property applies to addition and multiplication separately, and can be expressed with the following formulas:

## $』$ FORMULA TO KNOW

Associative Property of Addition

$$
(a+b)+c=a+(b+c)
$$

## Associative Property of Multiplication

$$
(a b) c=a(b c)
$$

```
A EXAMPLE
```

$(3+4)+6=3+(4+6)$
$4 \times(2 \times 8)=(4 \times 2) \times 8$

The associative property allows us to group addends or group factors in different ways. This is particularly helpful in mental math, where we might easily recognize that $4+6$ is 10. In such cases, regrouping can help us recognize certain sums or products to make mental math easier.

## - TERM TO KNOW

## Associative Property

A property of addition that allows terms to be grouped in any order; a property of multiplication that allows factors to be grouped in any order.

## 5. Distributive Property

The distributive property applies multiplication over addition. This property is applied when we have a factor multiplied by a sum. It got its name from the process of distributing the outside factor into each part of the sum. Here is the general rule of the distributive property:
$』$ FORMULA TO KNOW
Distributive Property

$$
a(b+c)=a b+a c
$$

```
\(\Leftrightarrow\) EXAMPLE
\begin{tabular}{rl}
\(2(4+3)\) & Distribute 2 into 4 and 3, separately \\
\((2 \times 4)+(2 \times 3)\) & Multiply inside the parentheses \\
\(8+6\) & Add \\
14 & Our solution
\end{tabular}
                                Our solution
```


## $\square$ HINT

The distributive property is especially useful when working with variables. Realistically, an easier approach with numerical examples is to evaluate what is inside the parentheses first, and then multiply the outside factor. (This also follows the Order of Operations).

$$
2(4+3)
$$

2(7)
14

However, as we work with algebraic expressions containing variables, the distributive property is going to be very helpful.

## - TERM TO KNOW

Distributive Property
A property of multiplication that states that a sum multiplied by a factor can be expressed as a sum of the products of each individual addend and that factor.

## SUMMARY

There are many different properties for addition and multiplication. We looked at an identity property, an inverse property, a commutative property, the associative property. Also, for multiplication, there is the distributive property.

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## 日 TERMS TO KNOW

## Associative Property

A property of addition that allows terms to be grouped in any order; a property of multiplication that allows factors to be grouped in any order.

## Commutative Property

A property of addition that allows terms to be added in any order; a property of multiplication that allows factors to be multiplied in any order.

## Distributive Property

A property of multiplication that states that a sum multiplied by a factor can be expressed as a sum of the products of each original addend and that factor.
$\leftrightharpoons$ FORMULAS TO KNOW

Associative Property of Addition

$$
(a+b)+c=a+(b+c)
$$

## Associative Property of Multiplication

$$
(a b) c=a(b c)
$$

## Commutative Property of Addition

$$
a+b=b+a
$$

## Commutative Property of Multiplication

$$
a b=b a
$$

Distributive Property

$$
a(b+c)=a b+a c
$$

## Identity Property of Addition

$$
a+0=a
$$

Identity Property of Multiplication

$$
a \cdot 1=a
$$

Inverse Property of Addition

$$
a+(-a)=0
$$

Inverse Property of Multiplication

$$
a \times \frac{1}{a}=1
$$

