Properties of Exponents

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WHAT'S COVERED

In this lesson, you will learn how to simplify an expression using the properties of exponents. Specifically, this lesson will cover:

1. Properties of Exponents

Problems with exponents can often be simplified using a few basic exponent properties. Exponents represent repeated multiplication. We will use this fact to discover important properties.

DID YOU KNOW

The word exponent comes from the Latin "expo" meaning "out of" and "ponere" meaning "place." While there is some debate, it seems that the Babylonians living in present-day Iraq were the first to do work with exponents (dating back to the 23rd century BC or earlier!)

1a. Product Property of Exponents

Let's take a look at the following example and how we can rewrite the exponents as a multiplication problem.

EXAMPLE

a^3a^2	Expand exponents to multiplication problem
$(aaa)(aa)$	Now we have $5\ a's$ being multiplied together

a^5 Our Solution

A quicker method to arrive at our answer would have been to just add the exponents. This is known as the **product property of exponents**.

FORMULA TO KNOW

Product Property of Exponents

$a^m \bullet a^n = a^{m+n}$

The important thing here is that the expressions must have the same base. If exponential expressions with the same base are multiplied together, we can simply add the exponents.

EXAMPLE

$3^2 \cdot 3^6 \cdot 3$ Same base, add the exponents $2+6+1$

3^9 Our Solution

1b. Quotient Property of Exponents

Rather than multiplying, we will now try to divide with exponents.

EXAMPLE

$\frac{a^5}{a^2}$ Expand Exponents

$\frac{aaaaa}{aa}$ Divide out two of the $a's$

aaa Convert to Exponents

a^3 Our Solution

A quicker method to arrive at the solution would have been to just subtract the exponents. This is known as the **quotient property of exponents**:

FORMULA TO KNOW

Quotient Property of Exponents

$\frac{a^m}{a^n} = a^{(m-n)}$

Just like with the product property, it is important to note that it only holds true when the bases are the same.

EXAMPLE

$\frac{7^{13}}{7^5}$ Same base, subtract the exponents $13-5$

7^8 Our Solution

1c. Power of a Power Property of Exponents

A third property we will look at will have an exponent raised to another exponent. This is investigated in the following example:

EXAMPLE

$(a^2)^3$	This means we have a^2 three times
$a^2 \cdot a^2 \cdot a^2$	Add exponents
a^6	Our Solution

A quicker method to arrive at the solution would have been to just multiply the exponents. This is known as the **power of a power property of exponents**.

FORMULA TO KNOW

Power of a Power Property of Exponents
$(a^m)^n = a^{mn}$

This property is often combined with two other properties: power of a product, and power of a quotient. We will look at these properties next.

1d. Power of a Product Rule

What happens when you have more than one factor being multiplied together and raised to a power?

EXAMPLE

$(ab)^3$	This means we have (ab) three times
$(ab)(ab)(ab)$	Three $a's$ and three $b's$ can be written with exponents
a^3b^3	Our Solution

A quicker method to arrive at the solution would have been to take the exponent of three and put it on each factor in parentheses. This is known as the **power of a product property of exponents**.

FORMULA TO KNOW

Power of a Product Property of Exponents
$(ab)^m = a^m b^m$

HINT

It is important to be careful to only use the power of a product property with multiplication inside parentheses. This property does NOT work if there is addition or subtraction.

$$(a+b)^m \neq a^m + b^m$$

These are NOT equal. Beware of this error!

1e. Power of a Quotient Property of Exponents

Now, what about when you are dividing terms and that whole set is raised to a power?

EXAMPLE

$\left(\frac{a}{b}\right)^3$	This means we have the fraction three times
$\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)$	Multiply fractions across the top and bottom, using exponents
$\frac{a^3}{b^3}$	Our Solution

A quicker method to arrive at the solution would have been to put the exponent on every factor in both the numerator and denominator. This is known as the **power of a quotient property of exponents**.

FORMULA TO KNOW

Power of a Quotient Property of Exponents

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

2. Putting It All Together

The power of a power, product, and quotient properties of exponents are often used together to simplify expressions.

EXAMPLE

$(a^5)^3a^2$	Use the Power of a Power Property to multiply 5 and 3
$a^{5\cdot 3}a^2$	Evaluate the multiplication in the exponent
$a^{15}a^2$	Use the Product Property to add the exponents 15 and 2
a^{15+2}	Evaluate the addition in the exponent
a^{17}	Our Solution

EXAMPLE

$\frac{(b^7)^2}{b^6}$	Use the Power of a Power Property to multiply 7 and 2
$\frac{b^{7\cdot 2}}{b^6}$	Evaluate the multiplication in the exponent
$\frac{b^{14}}{b^6}$	Use the Quotiet Property to subtract the exponents 14 and 6

b^{14-6} Evaluate the subtraction in the exponent

b^8 Our Solution

BIG IDEA

Rules of Exponents	Formula
Product Rule of Exponents	$a^m a^n = a^{m+n}$
Quotient Rule of Exponents	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Rule of Exponents	$(a^m)^n = a^{mn}$
Power of a Product Rule of Exponents	$(ab)^m = a^m b^m$
Power of a Quotient Rule of Exponents	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

SUMMARY

These five **properties of exponents** are often mixed up in the same problem. Often there is a bit of flexibility as to which property is used first. However, the order of operations still applies to a problem. For this reason, we suggest simplifying inside any parentheses first, then simplify any exponents (using power properties). Finally, simplify any multiplication or division (using product and quotient properties).

FORMULAS TO KNOW

Power of a Power Property of Exponents

$(a^n)^m = a^{nm}$

Power of a Product Property of Exponents

$(ab)^n = a^n b^n$

Power of a Quotient Property of Exponents

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Product Property of Exponents

$$a^n \cdot a^m = a^{n+m}$$

Quotient Property of Exponents

$$\frac{a^n}{a^m} = a^{n-m}$$