Properties of Fractional and Negative Exponents

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WHAT'S COVERED

In this lesson, you will learn how to simplify an expression with fractional exponents. Specifically, this lesson will cover:

1. Properties of Fractional and Negative Exponents
2. Using All Exponent Properties

1. Properties of Fractional and Negative Exponents

When we simplify radicals with exponents, we divide the exponent by the index. Another way to write division is with a fraction bar. This idea is how we will define rational exponents.

FORMULA TO KNOW

Properties of Fractional Exponents

Rule #1: $\sqrt[m]{a} = a^{\frac{1}{m}}$

Rule #2: $(\sqrt[m]{a})^n = a^{\frac{n}{m}}$

When converting between radical to exponent, the denominator of a rational exponent becomes the index on our radical. Likewise, the index on the radical becomes the denominator of the exponent. We can use this property to change any radical expression into an exponential expression.

EXAMPLE Notice how the index of the radical becomes the denominator of the fraction:

$$\left(\sqrt[5]{x}\right)^3 = x^{\frac{3}{5}}$$

$$\left(\sqrt[6]{3x}\right)^5 = (3x)^{\frac{5}{6}}$$

EXAMPLE Notice how the negative exponents come from reciprocals:

$$\frac{1}{\left(\sqrt[7]{a}\right)^3} = a^{-\frac{3}{7}}$$

$$\frac{1}{\left(\sqrt[3]{xy}\right)^2} = (xy)^{-\frac{2}{3}}$$

We can also change any rational exponent into a radical expression by using the denominator as the index.

EXAMPLE Again, note how the denominator of the exponent becomes the index of the radical:

$$a^{\frac{5}{3}} = \left(\sqrt[3]{a}\right)^5$$

$$(2mn)^{\frac{2}{7}} = \left(\sqrt[7]{2mn}\right)^2$$

EXAMPLE Again, note how the negative exponent means a reciprocal:

$$x^{-\frac{4}{5}} = \frac{1}{\left(\sqrt[5]{x}\right)^4}$$

$$(xy)^{-\frac{2}{9}} = \frac{1}{\left(\sqrt[9]{xy}\right)^2}$$

The ability to change between exponential expressions and radical expressions allows us to evaluate problems we had no means of evaluating before by changing to a radical.

EXAMPLE

$27^{-\frac{4}{3}}$	Change to radical, denominator is index, negative means reciprocal
$\frac{1}{\left(\sqrt[3]{27}\right)^4}$	Evaluate radical
$\frac{1}{(3)^4}$	Evaluate exponent
$\frac{1}{81}$	Our Solution

2. Using All Exponent Properties

The largest advantage of being able to change a radical expression into an exponential expression is we are now allowed to use all our exponent properties to simplify. The following table reviews all of our exponent properties.

BIG IDEA

Properties of Exponents	General Form
Product Property	$a^m a^n = a^{m+n}$
Quotient Property	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Property	$(a^m)^n = a^{mn}$
Power of a Product Property	$(ab)^m = a^m b^m$
Power of a Quotient Property	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
Zero Property of Exponents	$a^0 = 1$
Properties of Negative Exponents	$a^{-m} = \frac{1}{a^m}$ $\frac{1}{a^{-m}} = a^m$ $\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$

When adding and subtracting with fractions, we need to be sure to have a common denominator. When multiplying, we only need to multiply the numerators together and denominators together. The following examples show several different problems, using different properties to simplify rational exponents.

EXAMPLE

$a^{\frac{2}{3}} b^{\frac{1}{2}} a^{\frac{1}{6}} b^{\frac{1}{5}}$ Need common denominator on $a's$ (6) and $b's$ (10)

$a^{\frac{4}{6}} b^{\frac{5}{10}} a^{\frac{1}{6}} b^{\frac{2}{10}}$ Add exponents on $a's$ and $b's$

$a^{\frac{5}{6}} b^{\frac{7}{10}}$ Our Solution

EXAMPLE

$\left(x^{\frac{1}{3}}y^{\frac{2}{5}}\right)^{\frac{3}{4}}$ Multiply each exponent by $\frac{3}{4}$

$x^{\frac{1}{4}}y^{\frac{3}{10}}$ Our Solution

EXAMPLE

$\dfrac{x^2y^{\frac{2}{3}}\cdot 2x^{\frac{1}{2}}y^{\frac{5}{6}}}{x^{\frac{7}{2}}y^0}$ In numerator, need common denominator to add exponents

$\dfrac{2x^{\frac{5}{2}}y^{\frac{9}{6}}}{x^{\frac{7}{2}}}$ Subtract exponents on x, reduce exponents on y

$2x^{-1}y^{\frac{3}{2}}$ Negative exponent moves down to denominator

$\dfrac{2y^{\frac{3}{2}}}{x}$ Our Solution

EXAMPLE

$\left(\dfrac{x^{\frac{3}{2}}}{x^{\frac{1}{3}}}\right)^{\frac{1}{4}}$ Need common denominator on x's in parentheses; use (6)

$\left(\dfrac{x^{\frac{9}{6}}}{x^{\frac{2}{6}}}\right)^{\frac{1}{4}}$ Subtract exponents

$\left(x^{\frac{7}{6}}\right)^{\frac{1}{4}}$ Multiply $\frac{7}{6}$ by $\frac{1}{4}$

$x^{\frac{7}{24}}$ Our Solution

SUMMARY

It is important to remember that as we simplify with **fractional and negative exponents**, we are using the same properties we used when simplifying integer exponents. The only difference is we need to follow our rules for fractions as well. It may be worth reviewing your notes on exponent properties to be sure you're comfortable with using the properties.

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FORMULAS TO KNOW

Power of a Power Property of Exponents

$$(a^n)^m = a^{nm}$$

Power of a Product Property of Exponents

$$(ab)^n = a^n b^n$$

Power of a Quotient Property of Exponents

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Product Property of Exponents

$$a^n \cdot a^m = a^{n+m}$$

Properties of Fractional Exponents

Rule #1: $\sqrt[m]{a} = a^{1/m}$

Rule #2: $(\sqrt[m]{a})^n = a^{n/m}$

Quotient Property of Exponents

$$\frac{a^n}{a^m} = a^{n-m}$$