Properties of Logs

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to convert a logarithmic expression using the properties of logs. Specifically, this lesson will cover:

1. Product Property of Logs

When we learned about exponents, we learned that there are certain properties with exponents that can be applied when exponential expressions are multiplied, divided, and raised to exponent powers. There are similar properties with logarithms.

The **Product Property of Logarithms** allows us to split a logarithmic expression into several logs which are added together. More specifically, we break down the argument of the log (what is inside the parentheses) into factors. The logarithm is then applied to each individual factor, and the string of logs are added together. Below is the general property:

FORMULA TO KNOW

Product Property of Logs

$\log_b(xy) = \log_b(x) + \log_b(y)$

We can use this property to break a quantity down into more than two factors.

EXAMPLE Expand the logarithmic expression $\log_4(90)$.

$\log_4(90)$	Rewrite 90 with prime factorization

$\log_4(2\cdot 3\cdot 3\cdot 5)$	Apply the Product Property of Logs
$\log_4(2)+\log_4(3)+\log_4(3)+\log_4(5)$	Combine like terms
$\log_4(2)+2\log_4(3)+\log_4(5)$	Our solution

We can also use the Product Property in the other direction to simplify logarithmic expressions. Be sure that all of the logarithms have the same base, otherwise, we cannot use the property:

EXAMPLE Simplify the logarithmic expression $\log_5(2)+\log_5(3)+\log_5(7)$.

$\log_5(2)+\log_5(3)+\log_5(7)$	Apply the Product Property of Logs
$\log_5(2\cdot 3\cdot 7)$	Evaluate parentheses
$\log_5(42)$	Our solution

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2. Quotient Property of Logs

A property very similar to the product property exists with quotients. If we recognize a quotient within a logarithm, we can create individual logs connected with subtraction. The main difference here between the product and quotient properties is that the product property of logs connects individual logs with addition, and the **quotient property of logs** connects individual logs with subtraction.

FORMULA TO KNOW

Quotient Property of Logs

$$\log_b\left(\frac{x}{y}\right)=\log_b(x)-\log_b(y)$$

This property can be useful if there is a quotient explicitly written within a logarithm, especially if you can recognize a log that can be easily evaluated mentally:

EXAMPLE Expand the logarithmic expression $\log_4\left(\frac{16}{3}\right)$.

$\log_4\left(\frac{16}{3}\right)$	Apply the Quotient Property of Logs
$\log_4(16)-\log_4(3)$	Simplify $\log_4(16)=2$ because $4^2=16$
$2-\log_4(3)$	Our solution

We can also apply this property in the other direction:

EXAMPLE Simplify the logarithmic expression $\log_2(72)-\log_2(9)$.

$\log_2(72)-\log_2(9)$	Apply the Quotient Property of Logs

$\log_2\left(\frac{72}{9}\right)$	Evaluate parentheses
$\log_2(8)$	Simplify $\log_2(8) = 3$ because $2^3 = 8$
3	Our solution

3. Power Property of Logs

The **power property of logs** is useful when simplifying logarithmic expressions that contain an exponent within the operation. This property allows us to move the exponent outside of the log operation, and place it as a scalar multiplier to the log. In general, we write the power property as:

FORMULA TO KNOW

Power Product of Logs
$\log_b(x^n) = n \cdot \log_b(x)$

EXAMPLE Evaluate the logarithmic expression $\log_2(16)$.

$\log_2(16)$	Rewrite 16 as 2^4
$\log_2(2^4)$	Apply the Power Property of Logs
$4\log_2(2)$	Rewrite $\log_2(2)$ as 1
4	Our solution

HINT

As we saw in this example, if the argument of the log and the base are the same (both were 2 in the example above), the logarithm evaluates to 1. This relationship is explained later in this lesson.

4. Change of Base Property

The **change of base property** is extremely useful in using calculators to evaluate logarithmic expressions. Most calculators that can evaluate logs have only two buttons: a common log button, which is a base 10 log, and a natural log button, which is a base e log. e is a mathematical constant approximately equal to 2.718282.

What do we do, then, when we want to evaluate a log with base 3 using our calculator? Or a log with base 7? Or any other log with a base other than 10 or e? We use the change of base formula:

FORMULA TO KNOW

Change of Base Property of Logs

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Basically, we can take the common log or natural log of the argument, no matter what the base is, but then we must divide that expression by the common log or natural log (whichever was used for the numerator) by the original log's base. Here is a concrete example:

EXAMPLE Rewrite the logarithmic expression $\log_3(40)$ as a common log.

$\log_3(40)$	Apply Change of Base Property of Logs
$\frac{\log(40)}{\log(3)}$	Our solution

As we can see, $\log_3(40)$ on its own is difficult to evaluate using a calculator. However, thanks to the Change of Base formula, we can enter $\log(40) \div \log(3)$ into the calculator to evaluate the expression. This is helpful when given a problem like $\log_2(2)+\log_2(4)+\log_2(16)$. You can rewrite this and then use the log function on the calculator:

EXAMPLE Calculate the logarithmic expression $\log_2(2)+\log_2(4)+\log_2(16)$.

$\log_2(2)+\log_2(4)+\log_2(16)$	Apply Change of Base Property of Logs
$\frac{\log(2)}{\log(2)}+\frac{\log(4)}{\log(2)}+\frac{\log(16)}{\log(2)}$	Evaluate each log with a calculator
$1+2+4$	Add
7	Our solution

5. Other Logarithmic Properties

Finally, let's review two **other properties of logs** that can help us simplify expressions.

FORMULA TO KNOW

Other Properties of Logs

$\log_b(b) = 1$

$\log_b(1) = 0$

The first states that if the argument of the log and the base are the same, the expression evaluates to 1.

EXAMPLE $\log_5(5) = 1$

The last property states that the log of 1, no matter what the base is, evaluates to zero.

EXAMPLE $\log_3(1) = 0$ or $\log_7(1) = 0$

SUMMARY

There are many properties of logs that are useful when simplifying and solving logarithmic expressions and equations. The **product property of logs** allows us to rewrite the expression using addition, while the **quotient property of logs** allows us to rewrite the expression using subtraction. The **power property of logs** states that if there is an exponent within the operation, you can write this outside of the log expression as a scalar multiplier.

Common log and natural log are usually the only logs that can be calculated directly on the calculator. The **change of base** formula can be used for calculating logs with other bases. It is also important to note **other logarithmic properties**. If the argument of the log and the base are the same, the expression is equal to 1. The log of 1, no matter the base, equals zero.

FORMULAS TO KNOW

Change of Base Property of Logs

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Other Properties of Logs

$$\log_b(b) = 1$$

$$\log_b(1) = 0$$

Power Property of Logs

$$\log_b(x^n) = n \cdot \log_b(x)$$

Product Property of Logs

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

Quotient Property of Logs

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$