Quadratic Equations with No Real Solution

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to determine if a quadratic equation has real or non-real solutions by finding the value of the discriminant. Specifically, this lesson will cover:

1. The Discriminant of the Quadratic Formula

Working with the quadratic formula is one method in determining if there are no real solutions to a quadratic equation. Recall the quadratic formula:

FORMULA TO KNOW

Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The expression that is underneath the square root is called the discriminant. Because the discriminant is underneath a square root sign, it must not have a negative value, otherwise it does not evaluate to a real number. This is how we can tell if a quadratic has no real solutions by using the quadratic formula.

EXAMPLE Find the solutions for the quadratic equation $y=x^2-5x+8$.

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	Identify the values for a, b, and c in the equation $y=x^2-5x+8$
$a=1$, $b=-5$, $c=8$	Substitute these values in the quadratic formula

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(8)}}{2(1)}$$ In the discriminant, square -5 and multiply 4, 1, and 8

$$x = \frac{5 \pm \sqrt{25 - 32}}{2}$$ No real solutions

Because we have a negative number underneath the square root, we can conclude that this equation has no real solutions.

BIG IDEA

If the value of the discriminant is equal to 0 or greater than 0, then you're going to have a real solution for your quadratic equation. If the value of the discriminant is less than 0, then you're going to have a non-real solution for your quadratic equation.

2. Negative Square Roots and the Imaginary Unit

Even though some quadratic equations may have no real solutions, we can still express their solutions mathematically. To do so, we use the **imaginary number**, *i*, in the expression for its solution. The imaginary number, *i*, is a non-real number that represents the square root of -1.

FORMULA TO KNOW

Imaginary Number

$$i = \sqrt{-1}$$

The letter *i* is used to denote the square root of negative 1. We can rewrite the square roots of negative numbers using this letter.

EXAMPLE

$$\sqrt{-6} = \sqrt{6 \cdot -1} = \sqrt{6} \cdot \sqrt{-1} = \sqrt{6}i$$

$$\sqrt{-9} = \sqrt{9 \cdot -1} = \sqrt{9} \cdot \sqrt{-1} = 3i$$

$$\sqrt{-12} = \sqrt{4 \cdot 3 \cdot -1} = \sqrt{4} \cdot \sqrt{3} \cdot \sqrt{-1} = 2\sqrt{3} \cdot \sqrt{-1} = 2\sqrt{3}i$$

TERM TO KNOW

Imaginary Number

A non-real number that is expressed in terms of the square root of a negative number (usually the square root of -1, represented by i).

3. Complex Solutions to Quadratic Equations

If we encounter a negative value underneath the radical when using the quadratic formula, we can express the solutions to the quadratic equation using **complex numbers**. A complex number contains a real part and an imaginary part, such as $7+3i$ or $12-i$.

EXAMPLE Suppose we were calculating the solutions to a quadratic equation and got to this step:

$$x=\frac{6\pm\sqrt{-16}}{2}$$

This tells us that there are no real solutions since we cannot have a negative number in the discriminant. We can rewrite this result as complex numbers.

$x=\frac{6\pm\sqrt{-16}}{2}$	Rewrite square root
$x=\frac{6\pm\sqrt{16}\cdot\sqrt{-1}}{2}$	Evaluate the square root of 16 and -1
$x=\frac{6\pm 4i}{2}$	Create two separate solutions, one addition and one subtraction
$x=\frac{6-4i}{2},\ x=\frac{6+4i}{2}$	Divide each term by 2
$x=3-2i,\ x=3+2i$	Our solutions

In this example, the complex number $6\pm 4i$ was divided by 2. We can divide 6 and $4i$ by 2 separately to arrive at $3\pm 2i$. Then, we can create two expressions, one taking the minus sign, and the other taking the plus sign, due to the $\pm$ symbol.

TERM TO KNOW

Complex Number
A number containing a real component and an imaginary component.

SUMMARY

If the **discriminant of the quadratic formula** is greater than or equal to 0, then the solutions to the quadratic equation will be real numbers. If the discriminant is less than 0, the equation has no real solution. Looking at the graph of a quadratic equation, if the parabola does not cross or intersect the x-axis, then the equation has no real solution. No real solution does not mean that there is no solution, but that the solutions are not real numbers. **Negative square roots and the imaginary unit** are used to find **complex solutions to quadratic equations**.

Source: ADAPTED FROM "BEGINNING AND INTERMEDIATE ALGEBRA" BY TYLER WALLACE, AN OPEN SOURCE TEXTBOOK AVAILABLE AT www.wallace.ccfaculty.org/book/book.html. License: Creative Commons Attribution 3.0 Unported License

TERMS TO KNOW

Complex Number

A number in the form $a + bi$, containing a real part, a, and an imaginary part, *bi*, where *i* is the imaginary unit, $\sqrt{-1}$.

Imaginary Number

A non-real number that is expressed in terms of the square root of a negative number (usually the square root of -1, represented by *i*).

FORMULAS TO KNOW

Imaginary Number

$i = \sqrt{-1}$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

