Quadratic Equations with Non-Real Solutions

by Sophia

WHAT'S COVERED

This tutorial covers quadratic equations with non-real solutions, through the definition and discussion of:

1. Imaginary Numbers

You may recall that the square root of a negative number is non-real, because any real number squared will not be negative. The square root of -1 is defined as the **imaginary unit** ***i*** .

FORMULA TO KNOW

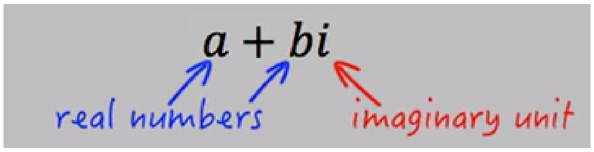
Imaginary Number

$\sqrt{-1} = i$

Imaginary numbers often arise when solving quadratic equations.

2. Complex Numbers

A complex number is a value in the following form, in which the variables *a* and *b* are real numbers and *i* is the imaginary unit.



Notice that in this standard form for writing a complex number, the real part *a* is written first and the imaginary part *bi* is written second.

> EXAMPLE In the complex number below, 5 is the *real* part of the complex number, and *3i* is the *imaginary* part. Furthermore, in the imaginary part, 3 is the coefficient and *i* is the imaginary unit.
>
> $5+3i$

Complex numbers can occur when solving a quadratic equation using the quadratic formula.

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3. The Quadratic Formula

When solving a quadratic equation set equal to zero, as shown below, the solution(s), x, to the quadratic equation can be found using the quadratic formula.

$$ax^2+bx+c=0$$

The variables *a*, *b*, and *c* in the **quadratic formula** correspond to the coefficients in the quadratic equation.

> FORMULA TO KNOW
>
> **Quadratic Formula**
>
> $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

If the expression under the square root is negative, then the quadratic equation will have zero real solutions. It follows, then, that when there are no real solutions to a quadratic equation, the graph of the equation will have zero x-intercepts, meaning that the parabola will never intersect the x-axis. In cases such as this, you can use the imaginary unit *i* to write the solutions of the quadratic equation as complex numbers.

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4. Solving Quadratic Equations with Non-Real Solutions

> EXAMPLE Suppose you want to solve the quadratic equation:
>
> $$x^2+16=0$$
>
> You would start by subtracting 16 from each side.

$$\begin{aligned} x^2 + 16 &= 0 \\ -16 \quad & -16 \\ x^2 &= -16 \end{aligned}$$

Next, you can cancel out the exponent by taking the square root on both sides.

$$\sqrt{x^2} = \sqrt{-16}$$
$$x = \pm\sqrt{-16}$$

HINT

Remember that you must include both the positive and negative solutions when taking the square root.

Now you can use the product property of radicals to write the square root of -16:

$$x = \pm\sqrt{16} \cdot \sqrt{-1}$$

The square root of 16 is 4, and the square root of -1 is *i*, so your solutions are:

$$x = \pm 4i$$

TRY IT

Consider the following quadratic equation:

$$x^2 + 5x + 8 = 0$$

Use the quadratic formula to solve this quadratic equation. +

The equation is in the form $ax^2 + bx + c = 0$, so we can use the quadratic formula. To use the formula, you need to identify your values for *a*, *b*, and *c*. Since x^2 has no written coefficients, you know that it has an implied coefficient of 1, which means *a* equals 1. You can also see that *b* equals 5, and *c* equals 8.

- $a = 1$
- $b = 5$
- $c = 8$

Substituting these values into the quadratic formula provides the expression below:

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(8)}}{2(1)}$$

Start by simplifying your denominator.

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(8)}}{2}$$

Next, simplify your numerator, starting with the expression underneath the square root.

$$x = \frac{-5 \pm \sqrt{25-32}}{2} = \frac{-5 \pm \sqrt{-7}}{2}$$

Did you notice that the expression underneath the square root is negative? This means that your solution will be non-real. You can use the product property of radicals to rewrite the square root of -7 as:

$$\sqrt{-7} = \sqrt{-1} \cdot \sqrt{7} = i\sqrt{7}$$

HINT

With imaginary numbers and radicals, you write the imaginary number first and then the radical.

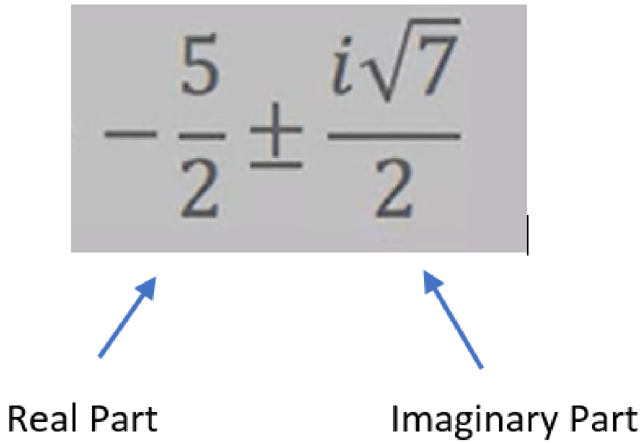
Therefore, your expression is:

$$x = \frac{-5 \pm i\sqrt{7}}{2}$$

Lastly, you can separate your solution into two parts, as shown below. You cannot simplify your fractions any further, because the numerator and denominator do not have any common factors other than 1; therefore, this is your final solution:

$$x = -\frac{5}{2} \pm \frac{i\sqrt{7}}{2}$$

Notice that your solution is a complex number in the form (a+bi), showing both the real part and the imaginary part.



SUMMARY

Today you reviewed **imaginary numbers**, recalling that the square root of a negative number is non-real because any real number squared will not be negative. You learned about **complex numbers**, which are values in the form a + bi, where a is the real part, and b times i is the imaginary part of the complex number. You also learned that when solving a quadratic equation using the **quadratic formula**, if the expression underneath a square root is negative, then the quadratic equation has zero real solutions. In cases such as this, when **solving quadratic equations with non-real solutions**, you learned that you can use the imaginary unit i to write the solutions of the quadratic equation as complex numbers.

Source: This work is adapted from Sophia author Colleen Atakpu.

FORMULAS TO KNOW

Imaginary Number

$$i = \sqrt{-1}$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$