Quadratic Factoring using the Difference of Squares

by Sophia

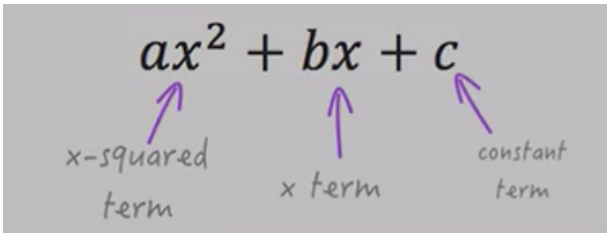
WHAT'S COVERED

This tutorial covers a specific type of quadratic factoring using the difference of squares, through the definition and discussion of:

1. Quadratic Expressions
2. Difference of Squares
3. Factoring Difference of Squares

1. Quadratic Expressions

A quadratic expression is a polynomial that can be written in the form shown below.



- The first term, ax^2, is called the x squared term, and the coefficient is a.
- The second term, bx, is called the x term, and the coefficient is b.
- The third term, c, is called the constant term.

2. Difference of Squares

A specific type of quadratic expression is called the difference of squares.

EXAMPLE You know that the expression shown here is a quadratic expression because the highest exponent in the expression is 2.

$x^2 - 16$

Notice that the constant term, 16, is a perfect square because the square root of 16 is 4, which is an integer. Therefore, you can rewrite your expression as the following:

$x^2 - 4^2$

This is called a difference of squares expression, because there are two values squared, x and the integer, with subtraction between the two.

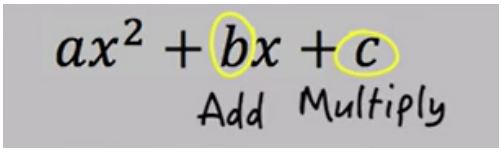
3. Factoring Difference of Squares

Difference of squares expressions can be factored similarly to other quadratic expressions.

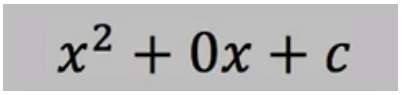
EXAMPLE Suppose you want to factor the difference of squares expression from the example above.

$x^2 - 16$

To factor, you need to find two numbers that multiply to the constant term and add to the coefficient of the middle term.



Note, when the x term is absent from a quadratic expression, the coefficient of the x term is 0. Therefore, you can write the x term with the coefficient of 0. However, because anything multiplied by 0 is 0, the entire term is 0, so in reality, you don't need to write anything for the x term.



You'll notice that in the expression that you want to factor, your constant term, 16, is a perfect square. Therefore, you can now write your expression as:

$x^2 - (4)^2$

To factor your expression, you need to find two numbers that add to 0 and multiply to -16.

HINT

It's important to note that when you factor, the only pair of numbers that sum to 0 are opposites of each other. Positive 1 and negative 1 sum to 0 and are opposites of each other. Negative 5 and positive 5 also sum to 0 and are opposites. Notice that the only difference between the numbers is their sign.

Going back to your expression, 4 and -4 are opposites, so they sum to 0, and they also multiply to negative 16. Therefore, you can factor your expression as:

$(x+4)(x-4)$

Notice that the integer being squared in the original expression, 4, is part of *both* factors: x plus 4 in the first factor, and x minus 4 in the second factor.

Therefore, in general, you can factor the difference of squares equations as shown here.

FORMULA TO KNOW

Difference of Squares

$a^2-b^2=(a+b)(a-b)$

TRY IT

Consider the expression:

$9x^2-25$

Use the difference of squares formula to factor this expression. +

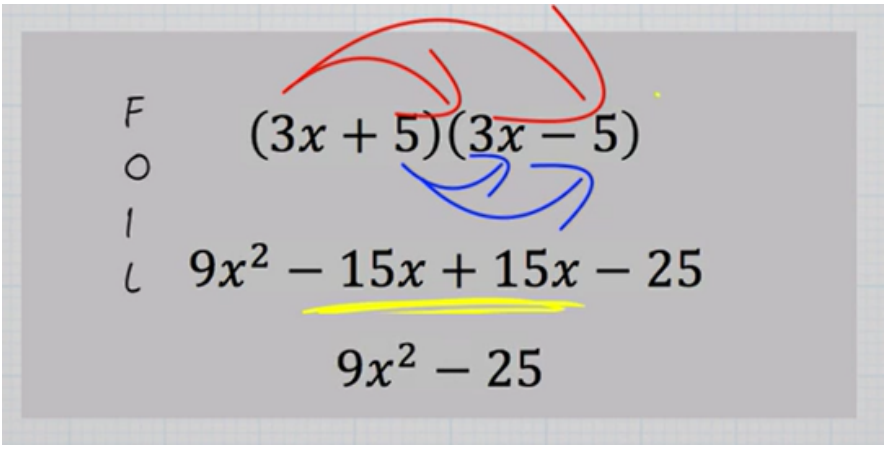
Notice that 9 and 25 are perfect squares. The square root of 9 is 3, and the square root of 25 is 5, so you can rewrite your expression as:

$(3x)^2-(5)^2$

Now you can use your formula to factor the expression. Your a term in the formula is 3x and your b term in the formula is 5.

$(3x+5)(3x-5)$

You can verify that you have factored the expression correctly by performing FOIL to see if you get your original expression.



Since you do indeed arrive back at your original expression, you have factored your expression correctly.

SUMMARY

Today you learned the definition of a **quadratic expression**, noting that the highest exponent in the expression is 2. You also learned about a specific type of quadratic expression called the **difference of squares**, in which there are two values squared, x and the integer, with subtraction between them. Lastly, you learned that the goal of **factoring difference of squares** is to find two numbers that multiply to the constant term, and add to the coefficient of the middle term. It's important to note that when factoring the difference of squares, the coefficient of the middle term is 0, and the only pair of numbers that sum to 0 are opposites of each other.

Source: This work is adapted from Sophia author Colleen Atakpu.

FORMULAS TO KNOW

Difference of Squares

$$a^2-b^2=(a+b)(a-b)$$