Quadratic Inequalities

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WHAT'S COVERED

In this lesson, you will learn how to determine the solution set to a quadratic inequality. Specifically, this lesson will cover:

1. Review of Inequalities

An inequality is a statement that two quantities are not equal to each other. In general, we see statements that use inequality symbols to show that one quantity is greater than or less than another. However, inequality symbols can be strict or non-strict. The distinction here is that non-strict inequalities "allow" the two quantities to be exactly equal to each other, while strict inequality symbols do not. This is the difference between "greater than" (>) and "greater than or equal to" (≥), for example.

It is also important to remember that when graphing inequalities and when plotting solutions on a number line, we use open circles, curved braces, and dotted lines for strict inequalities; and we use closed circles, square brackets, and solid lines for non-strict inequalities.

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2. Solutions to Quadratic Inequalities

When we find solutions to a quadratic inequality, we are looking for all x-values that make the inequality statement true. Let's take a look at an example.

> EXAMPLE Consider this quadratic inequality $2x^2+3x-7<4$.
>
> Solutions to this inequality are all x-values that make this inequality statement true. That is, there is a set of x-values that makes $2x^2+3x-7$ less than 4, and there is a set of x-values that makes $2x^2+3x-7$ greater than or equal to 4. The former is the solution set to the inequality, since it makes our statement true, while the latter is the set of all non-solutions because it makes the inequality statement false.

To find solutions to a quadratic inequality, it is often helpful to first think of the relationship as an equation, and then consider the inequality once solutions to the equation have been found. This is because we have a variety of tools at our disposal to solve quadratic equations, such as factoring, completing the square, or using the quadratic formula.

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3. Solving a Quadratic Inequality

We generally follow these steps to solve a quadratic inequality.

STEP BY STEP

1. Rewrite as an equation set equal to zero.
2. Solve the equation using which ever method you prefer (factoring, quadratic formula, or completing the square).
3. Use solutions to create intervals on a number line.
4. Choose a test value that falls within each interval on the number line.
5. Plug each test value into the inequality (with zero on one side) to identify solution regions.

In other words, when we treat the inequality as an equation and find solutions to the equation, we identify critical points to define the solution region. We then choose any value we want within certain intervals (defined by these critical points) and see if they yield true or false statements to the original inequality.

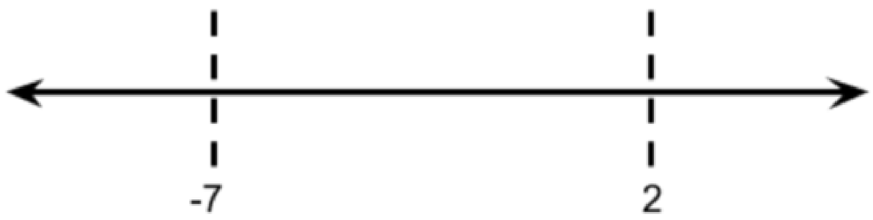
- If a test point satisfies the inequality, the interval it lies within is included in our solution region.
- If a test point does not satisfy the inequality, the interval it lies within is excluded from the solution region.

EXAMPLE Find the solutions for the quadratic inequality $x^2+5x-6>8$.

First, we need to write this as an equation and set it equal to zero. This means we will use an equals sign instead of an inequality symbol, and then subtract 8 from both sides. Then we can solve the quadratic equation:

$x^2+5x-6>8$	Rewrite as an equation set equal to zero
$x^2+5x-6=8$	Subtract 8 from both sides
$x^2+5x-14=0$	Factor the equation
$(x-2)(x+7)=0$	Set each factor equal to zero
$x-2=0,\quad x+7=0$	Evaluate
$x=2,\quad x=-7$	Solutions to the equation

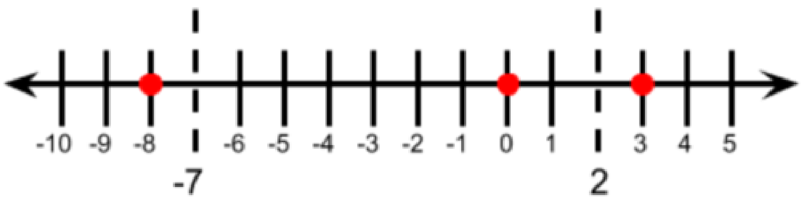
Now, use these solutions to create three intervals on the number line:



Next, we choose any value that fits within each interval. It doesn't matter which values we choose to be test values, but make it as simple as possible, like -8, 0, or 4. If possible, avoid using decimals. We are going to use them as x-values to be plugged into the inequality.

We can certainly plug these values into the original inequality that has 8 on one side of the inequality symbol. However, it makes the process easier to compare values to zero. This is because we just need to determine if the value is positive or negative to decide if it satisfies the inequality or not. We can rewrite the original inequality $x^2+5x-6>8$ as $x^2+5x-14>0$.

Using this rewritten inequality, choose test values within the three intervals. We need a test value that is less than -7, a test value that is between -7 and 2, and a test value greater than 2. Let's use -8, 0, and 4 and plug them into the inequality



x	$x^2+5x-14>0$	Result	Interval
-8	$(-8)^2+5(-8)-14>0$ $64-40-14>0$ $10>0$	This inequality is TRUE, so this interval is part of the solution.	$x<-7$
0	$0^2+5(0)-14>0$ $0+0-14>0$ $-14>0$	This inequality is NOT TRUE, so this interval is NOT part of the solution.	$-7\le x\le 2$
4	$4^2+5(4)-14>0$ $16+20-14>0$ $22>0$	This inequality is TRUE, so this interval is part of the solution.	$x>2$

The values $x<-7$ and $x>2$ are included in the solution region, while the values $-7\le x\le 2$ make up the non-solution region. Note that we used less than or greater than symbols with our solutions. This is because the original inequality $x^2+5x-6>8$ was a strict inequality, meaning that these points are not part of the inequality. We can write the solution to the inequality as:

$$x<-7 \text{ OR } x>2$$

SUMMARY

Recall that **inequalities** are statements that two quantities are not equal to each other. The**solution to quadratic inequalities** is a range of *x* values that make the inequality statement true. The process for **solving a quadratic inequality** is 1) solve as an equation set equal to 0; 2) use solutions to the equation to create intervals on a number line; 3) choose a test value that falls within each interval on the number line; and 4) plug each test value into the inequality to identify the solution regions.

