# Rationalizing the Denominator

*by Sophia*

WHAT'S COVERED

In this lesson, you will learn how to simplify algebraic fractions with a radical in the denominator. Specifically, this lesson will cover:

## 1. Rationalizing the Denominator

It is considered bad practice to have a radical in the denominator of a fraction. When this happens we multiply the numerator and denominator by the same thing in order to clear the radical. In the lesson on dividing radicals, we talked about how this was done with monomials. Here we will look at how this is done with binomials.

If the binomial is in the numerator the process to rationalize the denominator is essentially the same as with monomials The only difference is we will have to distribute in the numerator.

EXAMPLE

| | |
|---|---|
| $\frac{\sqrt{3}-9}{2\sqrt{6}}$ | Want to clear $\sqrt{6}$ in denominator; Multiply both numerator and denominator by $\sqrt{6}$ to clear the radical in the denominator |
| $\frac{\sqrt{3}-9}{2\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right)$ | Distribute the $\sqrt{6}$ through the numerator, multiply the denominator by $\sqrt{6}$ |
| $\frac{\sqrt{3}(\sqrt{6})-9(\sqrt{6})}{2\sqrt{6}(\sqrt{6})}$ | Evaluate any multiplication, $(\sqrt{6}(\sqrt{6})=6)$ |
| $\frac{\sqrt{18}-9\sqrt{6}}{2\cdot 6}$ | Simplify radicals in numerator ($\sqrt{18}$ can be broken down to $\sqrt{9\cdot 2}$), multiply out denominator |
| $\frac{\sqrt{9\cdot 2}-9\sqrt{6}}{12}$ | Take square root of 9 |
| $\frac{3\sqrt{2}-9\sqrt{6}}{12}$ | Reduce each term by 3 |
| $\frac{\sqrt{2}-3\sqrt{6}}{4}$ | Our Solution |

HINT

It is important to remember that when reducing the fraction we cannot reduce with just the 3 and 12 or just the 9 and 12. When we have addition or subtraction in the numerator or denominator we must divide all terms by the same number. As we are rationalizing it will always be important to constantly check our problem to see if it can be simplified more. We ask ourselves, can the fraction be reduced? Can the radicals be simplified? These steps may happen several times on our way to the solution.

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## 2. Using a Conjugate

If the binomial occurs in the denominator we will have to use a different strategy to clear the radical.

> ↪ EXAMPLE Consider $\frac{2}{\sqrt{3}-5}$. If we were to multiply the denominator by $\sqrt{3}$ we would have to distribute it and we would end up with $3-5\sqrt{3}$. We have not cleared the radical, only moved it to another part of the denominator. So our current method will not work.

Instead, we will use what is called a conjugate. A**conjugate** is made up of the same terms, with the opposite sign in the middle. So for our example with $\sqrt{3}-5$ in the denominator, the conjugate would be $\sqrt{3}+5$

The advantage of a conjugate is when we multiply them together, we have $(\sqrt{3}-5)(\sqrt{3}+5)$, which is a difference and a sum. If we multiply these, we get a difference of squares. The final value ends up being the square of $\sqrt{3}$ and the square of 5, with subtraction in the middle:

$$(\sqrt{3}-5)(\sqrt{3}+5)$$
$$\sqrt{3}(\sqrt{3})+5\sqrt{3}-5\sqrt{3}-5(5)$$
$$(\sqrt{3})^2-5^2$$
$$3-25$$
$$-22$$

Our answer when multiplying conjugates will no longer have a square root, which is exactly what we want.

> ☆ BIG IDEA
>
> To rationalize a denominator containing a radical expression, multiply the fraction using its conjugate. The product will no longer contain a radical.

↪ EXAMPLE

| | |
|---|---|
| $\frac{2}{\sqrt{3}-5}$ | Want to clear radical from denominator; Multiply numerator and denominator by conjugate, $\sqrt{3}+5$ |
| $\frac{2}{\sqrt{3}-5}\left(\frac{\sqrt{3}+5}{\sqrt{3}+5}\right)$ | Distribute numerator, difference of squares in denominator |
| $\frac{2\sqrt{3}+2(5)}{\sqrt{3}(\sqrt{3})-5(5)}$ | Evaluate multiplication in numerator and denominator |
| $\frac{2\sqrt{3}+10}{3-25}$ | Evaluate denominator |
| $\frac{2\sqrt{3}+10}{-22}$ | Simplify solution by dividing by 2 |

$$\frac{\sqrt{3}+5}{-11}$$ Our Solution

We could have reduced by dividing by -2 instead of 2, giving $\frac{-\sqrt{3}-5}{11}$. Both answers are correct.

↪ EXAMPLE

$$\frac{\sqrt{15}}{\sqrt{5}+\sqrt{3}}$$ Want to clear radicals from denominator; Multiply numerator and denominator by conjugate, $\sqrt{5}-\sqrt{3}$

$$\frac{\sqrt{15}}{\sqrt{5}+\sqrt{3}}\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)$$ Distribute numerator, difference of squares in denominator

$$\frac{\sqrt{15}(\sqrt{5})-\sqrt{15}(\sqrt{3})}{\sqrt{5}(\sqrt{5})-\sqrt{3}\sqrt{3}}$$ Evaluate multiplication in both numerator and denominator

$$\frac{\sqrt{75}-\sqrt{45}}{5-3}$$ Simplify denominator

$$\frac{\sqrt{75}-\sqrt{45}}{2}$$ Break down radicals: $\sqrt{75}$ can be broken down to $\sqrt{25\cdot 3}$, $\sqrt{45}$ can be broken down to $\sqrt{9\cdot 5}$

$$\frac{\sqrt{25\cdot 3}-\sqrt{9\cdot 5}}{2}$$ Take square roots where possible

$$\frac{5\sqrt{3}-3\sqrt{5}}{2}$$ Our Solution

The same process can be used when there is a binomial in the numerator and denominator. We just need to remember to FOIL out the numerator.

↪ EXAMPLE

$$\frac{3-\sqrt{5}}{2-\sqrt{3}}$$ Want to clear radicals from denominator; Multiply numerator and denominator by conjugate, $2+\sqrt{3}$

$$\frac{3-\sqrt{5}}{2-\sqrt{3}}\left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right)$$ FOIL in numerator, difference of squares in denominator

$$\frac{3(2)+3(\sqrt{3})-\sqrt{5}(2)-\sqrt{5}(\sqrt{3})}{2(2)+\sqrt{3}(\sqrt{3})}$$ Evaluate multiplication in numerator and denominator

$$\frac{6+3\sqrt{3}-2\sqrt{5}-\sqrt{15}}{4-3}$$ Simplify denominator

$$\frac{6+3\sqrt{3}-2\sqrt{5}-\sqrt{15}}{1}$$ Numerator cannot be simplified any further; Divide each term by denominator, 1

$$6+3\sqrt{3}-2\sqrt{5}-\sqrt{15}$$ Our Solution

↪ EXAMPLE

$$\frac{2\sqrt{5}-3\sqrt{7}}{5\sqrt{6}+4\sqrt{2}}$$ Want to clear radicals from denominator; Multiply numerator and denominator by conjugate, $5\sqrt{6}-4\sqrt{2}$

 

| | |
|---|---|
| $\dfrac{2\sqrt{5}-3\sqrt{7}}{5\sqrt{6}+4\sqrt{2}}\left(\dfrac{5\sqrt{6}-4\sqrt{2}}{5\sqrt{6}-4\sqrt{2}}\right)$ | FOIL in numerator, difference of squares in denominator |
| $\dfrac{2\sqrt{5}(5\sqrt{6})-2\sqrt{5}(4\sqrt{2})-3\sqrt{7}(5\sqrt{6})+3\sqrt{7}(4\sqrt{2})}{5\sqrt{6}(5\sqrt{6})-4\sqrt{2}(4\sqrt{2})}$ | Evaluate multiplication in numerator and denominator |
| $\dfrac{10\sqrt{30}-8\sqrt{10}-15\sqrt{42}+12\sqrt{14}}{25\cdot 6-16\cdot 2}$ | Evaluate multiplication in denominator |
| $\dfrac{10\sqrt{30}-8\sqrt{10}-15\sqrt{42}+12\sqrt{14}}{150-32}$ | Evaluate subtraction in denominator; Cannot be simplified any further |
| $\dfrac{10\sqrt{30}-8\sqrt{10}-15\sqrt{42}+12\sqrt{14}}{118}$ | Our solution |

DID YOU KNOW

During the 5th century BC in India, Aryabhata published a treatise on astronomy. His work included a method for finding the square root of numbers that have many digits.

TERM TO KNOW

**Conjugate**

The conjugate of a binomial is a binomial with the opposite sign between its terms.

## SUMMARY

**Rationalizing the denominator** involves multiplying by a conjugate in both the denominator and the numerator of a fraction and then simplifying. The reason that we do that is because having an irrational radical in the denominator of a fraction is not considered simplified. The conjugate of the $\sqrt{a}+b$ is just $\sqrt{a}-b$. We just use the opposite sign in between the two terms. The conjugate of just a radical by itself is that same radical.



## TERMS TO KNOW

**Conjugate**

The conjugate of a binomial is a binomial with the opposite sign between its terms.