Set Notation and Interval Notation

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WHAT'S COVERED

In this lesson, you will learn how to identify the correct notation for a given number line. Specifically, this lesson will cover:

1. Interval Notation

In the previous lesson, we learned how to graph inequalities, but how could we write a specific interval without showing a picture on the number line? We need a few more symbols to express whether the endpoint is included within the interval or not. These include square brackets and parentheses.

Symbols for Notating Intervals

Endpoint is Included in Interval	Endpoint is Not Included in Iinterval
[or]	(or)
	

When writing an inequality using interval notation, square brackets correspond to the closed circles on the number line. They indicate that the endpoint is included in the interval. The parentheses or rounded brackets are used when there are open circles on the number line, and the endpoint is not included in the interval.

EXAMPLE Write the highlighted range on the number line in interval notation.

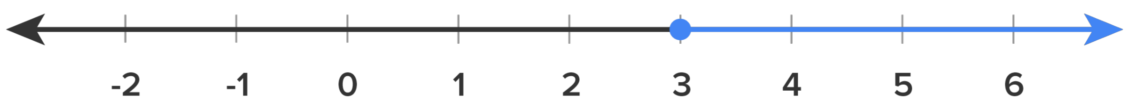
First, we note the endpoints of the highlighted range. There is an endpoint (or circle) at -2 and at 3.

These will be the numbers written in our interval.

We will just need to put the correct bracket in front of the -2 and behind the 3. We see that there is an open circle at -2; this will correspond to a parenthesis in front of the -2. There is a closed circle at the positive 3. This will correspond to a square bracket after the 3.

The interval notation for this highlighted range is (-2, 3].

↪ EXAMPLE Write the highlighted range on the number line in interval notation.



For this example, we will look at the endpoints of the highlighted interval. There is an endpoint at 3. Notice that the line for the interval simply has an arrow that points in the positive direction. If there is an arrow and not another circle on the interval, this means that the interval extends all the way to infinity. We write the infinity symbol as ∞. If the arrow is pointing in the positive direction, this is ∞ (positive infinity). If the arrow is pointing in the negative direction, this is $-\infty$ (negative infinity). Numbers can get bigger and bigger, but we will never actually reach ∞. So, ∞ is an endpoint of the interval, but is not included in the interval. Therefore, we will always use a parenthesis with positive or negative infinity.

In this example, 3 is an endpoint with a closed circle, which is written as $[3,$. The next endpoint is positive infinity, which is written as $\infty)$ with a parenthesis.

The interval notation for this highlighted range is $[3, \infty)$.

BIG IDEA

Interval notation describes a range of values from a starting point to a stopping point. We use either square brackets or parentheses, depending on if we are including or excluding exact values of the endpoints. Square brackets [ ] are used to include endpoints, and parentheses ( ) are used to exclude endpoints. We always use parentheses with the infinities.

2. Set Notation

When looking at number lines with a range of highlighted values, we can write the range of values in a couple of different ways: in a format called set notation, and in interval notation. We will use inequalities to help us write the intervals in set notation. Let's look at the symbols so far and add in the inequality symbols also.

Endpoint Symbols for Notating Intervals

Endpoint is Included in Interval	Endpoint is Not Included in Interval
●	○
[or]	(or)
$\leq$ or $\geq$	$<$ or $>$

If we add inequalities to the chart of endpoint symbols, we see that the less than (<) and the greater than (>) symbols indicate that the endpoint is not included in the interval. We see that the less than or equal to (≤) or greater than or equal to (≥) symbols do include the endpoints in the interval. If we have an expression such as x greater than or equal to −2, this means our variable can be any number greater than or equal to −2, such as 5, 0, −1, −1.9999, or even −2.

In set notation, we can use inequalities to help us write the interval in set notation. In set notation, we define the range of values as a set of numbers, and we use curly brackets to define the set, with a description of what is to be included in the set.

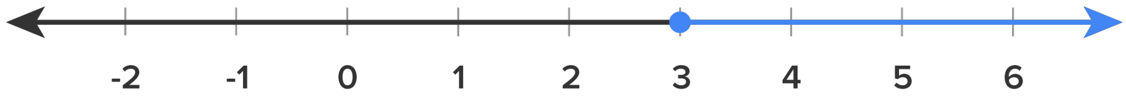
↱ EXAMPLE Write the highlighted range in set notation.

We see that the highlighted values range has endpoints at -2 to +3. There is an open circle at -2 which corresponds to the greater than symbol because -2 is not included in the interval. Also, there is 3, which corresponds to a less than or equal to sign, because the interval goes up to 3 and can equal 3 also. We can write this range of values as the interval where x is between -2 and 3. In set notation, we write $\{x \mid 2 < x \leq 3\}$. This reads as "all x-values, such that x is greater than (but not including) -2, but x is also less than and can include 3."

So, that means that x can be any number in the highlighted range that is between the values of -2 and 3, but x cannot equal -2 and can equal any value up to and including 3.

The set notation for this highlighted range is $\{x \mid 2 < x \leq 3\}$.

↱ EXAMPLE Write the highlighted range in set notation.



We first determine our endpoints. There is only one endpoint at 3 with a closed circle. The line extends to positive infinity. Let's think about the *x* values that would be included in this interval. In this interval, this would be any number greater than 3 or equal to 3. We would use the greater than or equal to symbol. In set notation, we write $\{x \mid x \geq 3\}$. This is read as, "all x-values, such that *x* is greater than or equal to 3." So, that means that *x* can be any number in the highlighted range that is greater than or equal to 3.

The set notation for this highlighted range is $\{x \mid x \geq 3\}$.

BIG IDEA

Set notation uses curly braces to define the number line solution as a set of values. Open circles correspond to the strict inequalities < and >, while closed circles correspond to non-strict inequalities ≤ and ≥. In set notation, the vertical bar is read, "such that...".

3. Writing a Solution in Interval and Set Notation

Let's practice writing in set and interval notation, using number line solutions.

EXAMPLE Examine the number line below:

We notice that all values from 2 to positive infinity are highlighted.

- In set notation, we would write this as $\{x \mid x > 2\}$. Note that our inequality symbol does not include the exact value of 2.
- In interval notation, this is written as $(2, \infty)$. Note that we use a parenthesis to the left of 2, and also a parenthesis to the right of infinity.

Let's look at a more complicated example:

EXAMPLE Examine the number line below:

This number line has two ranges highlighted. How do we write this in set notation and interval notation? We have two inequality statements to include in our set. First, we have $x < -4$. We also have $x \geq 1$.

- In set notation, we write $\{x \mid x < -4 \text{ or } x \geq 1\}$. We use the connecting word "or" because values that fit within either inequality statement will fit the number line solution.
- In interval notation, we define two intervals: the first interval is $(-\infty, -4)$, and the second interval is $[1, \infty)$. To accept both intervals as solutions for x, we use the symbol for union, U, to connect the two intervals. To complete our solution in interval notation, we have $(-\infty, -4) \cup [1, \infty)$.

SUMMARY

In **set notation**, we define the range of values as set of numbers and use curly braces to define the set, with a description of what is to be included in the set. **Interval notation**, on the other hand, describes an interval of values represented by the number line and use parentheses or brackets. When **writing a solution in interval and set notation**, it is important to pay attention to the different inequality signs and corresponding symbols.