Simplifying Radical Expressions

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WHAT'S COVERED

In this lesson, you will learn how to simplify a given radical expression. Specifically, this lesson will cover:

1. Simplifying Radical Expressions

Not all numbers have a nice even square root. For example, if we found $\sqrt{8}$ on our calculator, the answer would be 2.828427124746190097603377448419... and even this number is a rounded approximation of the square root. Decimal approximations are good estimates, but you may need the exact value. To do this, we will simplify radical expressions using the **product property of square roots**:

FORMULA TO KNOW

Product Property of Square Roots

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

We can use the product rule to simplify an expression such as $\sqrt{180}$ by splitting it into two roots, $\sqrt{36} \cdot \sqrt{5}$ and simplifying the first root to get $6\sqrt{5}$. The trick in this process is being able to translate a problem like $\sqrt{180}$ into $\sqrt{36} \cdot \sqrt{5}$. There are several ways this can be done.

The most common and, with a bit of practice, the fastest method, is to find perfect squares that divide evenly into the radicand, or number under the radical.

EXAMPLE

$\sqrt{75}$	75 is divisible by 25, a perfect square
$\sqrt{25 \cdot 3}$	Split into factors

$\sqrt{25} \cdot \sqrt{3}$ Product rule, take the square root of 25

$5\sqrt{3}$ Our Solution

If there is a coefficient in front of the radical to begin with, the problem merely becomes a big multiplication problem.

EXAMPLE

$5\sqrt{63}$ 63 is divisible by 9, a perfect square

$5\sqrt{9 \cdot 7}$ Split into factors

$5\sqrt{9} \cdot \sqrt{7}$ Product rule, take the square root of 9

$5 \cdot 3\sqrt{7}$ Multiply coefficients

$15\sqrt{7}$ Our Solution

As we simplify radicals using this method it is important to be sure our final answer can be simplified no more.

EXAMPLE

$\sqrt{72}$ 72 is divisible by 9, a perfect square

$\sqrt{9 \cdot 8}$ Split into factors

$\sqrt{9} \cdot \sqrt{8}$ Product rule, take the square root of 9

$3\sqrt{8}$ But 8 is also divisible by a perfect square, 4

$3\sqrt{4 \cdot 2}$ Split into factors

$3\sqrt{4} \cdot \sqrt{2}$ Product rule, take the square root of 4

$3 \cdot 2\sqrt{2}$ Multiply

The previous example could have been done in fewer steps if we had noticed that $76 = 36 \cdot 2$, but often the time it takes to discover the larger perfect square is more than it would take to simplify in several steps.

1a. Product Property

We can simplify higher roots in much the same way we simplified square roots, using the **product property of radicals**.

FORMULA TO KNOW

Product Property of Radicals

$\sqrt[m]{ab} = \sqrt[m]{a} \cdot \sqrt[m]{b}$

Often we are not as familiar with higher powers as we are with squares. It is important to remember what index we are working with as we try and work our way to the solution.

EXAMPLE

$\sqrt[3]{54}$	We are working with a cubed root, want third powers
$2^3=8$	Test 2, $2^3=8$, 54 is not divisible by 8
$3^3=27$	Test 3, $3^3=27$, 54 is divisible by 27!
$\sqrt[3]{27\cdot 2}$	Write as factors
$\sqrt[3]{27}\cdot\sqrt[3]{2}$	Product rule, take cubed root of 27
$3\sqrt[3]{2}$	Our Solution

Just as with square roots, if we have a coefficient, we multiply the new coefficients together.

EXAMPLE

$3\sqrt[4]{48}$	We are working with a fourth root, want fourth powers
$2^4=16$	Test 2, $2^4=16$, 48 is divisible by 16!
$3\sqrt[4]{16\cdot 3}$	Write as factors
$3\sqrt[4]{16}\cdot\sqrt[4]{3}$	Product rule, take fourth root of 16
$3\cdot 2\sqrt[4]{3}$	Multiply coefficients
$6\sqrt[4]{3}$	Our Solution

1b. Quotient Property

Now let's talk about the quotient property of radicals, which states that if you have the nth root of a and you divide that by the nth root of *b*, you can write this as a single quotient, $\frac{a}{b}$, and then take the nth root of that. Notice again that our roots must be the same.

FORMULA TO KNOW

Quotient Property of Radicals

$$\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

EXAMPLE

$\frac{\sqrt{18}}{\sqrt{2}}$	Use quotient property of radicals
$\sqrt{\frac{18}{2}}$	Divide 18 by 2
$\sqrt{9}$	Simplify
3	Our Solution

Let's try another example going in the other direction.

EXAMPLE

$\sqrt[3]{\frac{27}{8}}$	Use quotient property of radicals
$\frac{\sqrt[3]{27}}{\sqrt[3]{8}}$	Since 27 and 8 are both perfect cubes, take the cubed root of the numerator and denominator
$\frac{3}{2}$	Our Solution

SUMMARY

We can use the product or quotient properties to combine or break down radicands through multiplication or division and this will help us **simplify radical expressions**. When we're combining two radicals into one using the product or quotient property, the index of the radicals must be the same.

FORMULAS TO KNOW

Product Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Product Property of Square Roots

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

Quotient Property of Radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$