Simplifying Rational Expressions

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WHAT'S COVERED

In this lesson, you will learn how to simplify a quadratic rational expression by factoring and canceling like terms. Specifically, this lesson will cover:

1. Simplifying Numeric Fractions

When first learning how to simplify rational expressions (or algebraic fractions), it can be very helpful to review how we simplify numeric fractions (containing no variables). This is because the thought process is the same. The only difference is that we have variables and algebraic factors to consider when simplifying rational expressions.

EXAMPLE Simplify the fraction $\frac{12}{16}$.

To simplify this fraction, we break both the numerator and denominator down into their prime factors. From there, we see what factors appear in both the numerator and denominator, and remove them from the fraction completely. What we are left with is the simplified fraction:

$\frac{12}{16}$	Break numerator and denominator into prime factors
$\frac{2\cdot2\cdot3}{2\cdot2\cdot2\cdot2}$	Two factors of 2 cancel
$\frac{\cancel{2}\cdot\cancel{2}\cdot3}{\cancel{2}\cdot\cancel{2}\cdot2\cdot2}$	Simplify
$\frac{3}{2\cdot2}$	Evaluate
$\frac{3}{4}$	Our simplified fraction

2. Simplifying Rational Expressions with Factored Polynomials

The same principle applies to simplifying rational expressions. We look for common factors in both the numerator and denominator and cancel them. The only tricky there here is identifying those common factors, and in some cases, not confusing them for terms.

EXAMPLE Simplify the expression $\frac{(x+1)(x-2)}{(x-2)(x+3)}$.

$\frac{(x+1)(x-2)}{(x-2)(x+3)}$	The factors $(x-2)$ cancel
$\frac{(x+1)\cancel{(x-2)}}{\cancel{(x-2)}(x+3)}$	Simplify
$\frac{x+1}{x+3}$	Our solution

HINT

Be careful here. Many people make the mistake of thinking that x is a common factor. Here, x is a term that is part of two entirely different factors. We cannot cancel part of a factor, we can only cancel out entire, complete factors. In other words, we cannot simplify the above fraction to 1/3.

3. Simplifying Rational Expressions by Factoring

Simplifying rational expressions would be so easy if all rational expressions were written in factored form. Unfortunately, this isn't the case. However, we may be able to write the numerator and denominator as factors, or at least factor out a few common factors, in order to cancel and simplify the expression.

EXAMPLE Simplify the expression $\frac{2x^2+6x+6}{4x^2+4x-48}$.

One strategy is to see if there is a common factor between all terms of the numerator, and a common factor between all terms in the denominator. Here, we see that a 2 can be factored out of each term in the numerator. We can also factor out a 4 in all of the terms in the denominator. Let's see how this helps us simplify:

$\frac{2x^2+6x+6}{4x^2+4x-48}$	Factor out 2 in the numerator and 4 in the denominator
$\frac{2(x^2+3x+3)}{4(x^2+x-12)}$	Cancel 2 out of both numerator and denominator
$\frac{\cancel{2}(x^2+3x+3)}{\cancel{2}\cdot 2(x^2+x-12)}$	Simplify
$\frac{x^2+3x+3}{2(x^2+x-12)}$	Distribute denominator
$\frac{x^2+3x+3}{2x^2+2x-24}$	Our simplified fraction

HINT

Even if we were to factor the numerator and denominator, there would be no more common factors, so we have found the simplified fraction.

Let's work through a final example, in which factoring the expressions in the numerator and denominator will lead to common factors:

EXAMPLE Simplify $\frac{6x^2-18x-60}{4x^2-4x-24}$.

$\frac{6x^2-18x-60}{4x^2-4x-24}$	Factor out 6 in the numerator and 4 in the denominator
$\frac{6(x^2-3x-10)}{4(x^2-x-6)}$	6/4 simplifies to 3/2
$\frac{3(x^2-3x-10)}{2(x^2-x-6)}$	Factor the numerator and denominator
$\frac{3(x+2)(x-5)}{2(x+2)(x-3)}$	The factors $(x+2)$ cancel
$\frac{3(x-5)}{2(x-3)}$	Distribute both numerator and denominator
$\frac{3x-15}{2x-6}$	Our simplified function

HINT

You may choose to leave the fraction expression in factored form $\frac{3(x-5)}{2(x-3)}$ or in standard form $\frac{3x-15}{2x-6}$. Both are considered fully simplified because all common factors have been canceled.

SUMMARY

Recall that when **simplifying numeric fractions**, you need to break both the numerator and denominator down into its prime factors. A rational expression is a fraction in which the numerator and denominator are polynomials. To **simplify rational expressions with factored polynomials**, cancel any common factors. factor the expressions in the numerator and denominator. For more complex examples, you may need to first **simplify rational expressions by factoring**, then cancel any common factors.