Solving a Quadratic Equation by Factoring

by Sophia

WHAT'S COVERED

This tutorial covers how to solve a quadratic equation by factoring, through the exploration of:

1. Solutions to Quadratic Equations
2. The Zero Product Principle
3. Solving Quadratic Equations by Factoring

1. Solutions to Quadratic Equations

An equation is a mathematical statement that two quantities have the same value. An equal sign between the two quantities is used to show that they are equal.

Quadratic equations can be written in the following form, in which the variables a, b, and c are real numbers. When quadratic equations are set to 0, the solutions are the values for x that make the expression equal to 0. Therefore, the solutions are commonly referred to as zeroes or roots.

FORMULA TO KNOW

Quadratic Equation

$$ax^2 + bx + c = 0$$

Quadratic equations are set equal to 0 in order to solve them using different methods, including factoring and the quadratic formula. This lesson focuses on solving quadratic equations by factoring.

2. The Zero Product Principle

Solving quadratic equations by factoring uses the zero product principle.

KEY CONCEPT

The zero product principle states that if a product of two factors is zero, then one of the factors must be zero.

This is because anything or any value x multiplied by 0 equals 0. So, zero can be expressed as the product of zero in any other real number x.

$$x \cdot 0 = 0$$
$$0 = 0 \cdot x$$

HINT

The zero product principle can only be used when one side of the quadratic equation is equal to 0.

EXAMPLE Suppose you want to solve the following quadratic equation. The zero product principle tells you that solutions to this equation exist when x plus 7 equals 0 and when x minus 4 equals 0.

$$(x+7)(x-4)=0$$
$$x+7=0$$
$$x-4=0$$

Therefore, you can solve each of these equations to find your two solutions. Solving the first equation, start by subtracting 7 on both sides, which gives you x equals -7 for your first solution. For the second equation, add 4 to both sides, which gives you x equals 4. This means that the quadratic equation has two solutions: x equals -7 and x equals 4.

$$\begin{array}{rcl} x+7=0 & \quad & x-4=0 \\ -7 \ \ -7 & & +4 \ \ +4 \\ x=-7 & & x=4 \end{array}$$

3. Solving Quadratic Equations by Factoring

You can use factoring and the zero product principle to find solutions to quadratic equations.

EXAMPLE Suppose you want to solve the following quadratic equation by factoring:

$$x^2+7x+10=0$$

If you can factor the quadratic expression on the left side of the equation, then you can use the zero product principle to find solutions. Therefore, you want to find two numbers that multiply to 10 and add to 7.

Start by listing the pairs of numbers that multiply to 10, then find the pair of numbers that also adds to 7.

Factors of 10	Add to 7
1, 10 ✘	$1+10=11$

-1, -10 ✘	$(-1)+(-10)=-11$
2, 5 ✔	$2+5=7$
-2, -5	

Now you can rewrite your expression in factored form as the following, which will be equal to 0.

$$(x+2)(x+5)=0$$

Next, set each factor equal to 0 to write two separate equations, and solve accordingly, to arrive at your two solutions, x equals -2 and x equals -5.

$$\begin{array}{ll} x+2=0 & x+5=0 \\ \;\;-2\;\;-2 & \;\;-5\;\;-5 \\ \;\;x=-2 & \;\;x=-5 \end{array}$$

You can verify each solution by substituting it back into the original equation and simplifying. Substituting your first solution of -2 into the equation provides:

$$\begin{aligned} &(-2)^2+7(-2)+10=0 \\ &4-14+10=0 \\ &0=0 \end{aligned}$$

Your final equation is 0 equals 0, which is a true statement, so x equals -2 is correct.

Substituting -5, your second solution, into the equation provides:

$$\begin{aligned} &(-5)^2+7(-5)+10=0 \\ &25-35+10=0 \\ &0=0 \end{aligned}$$

Again, 0 equals 0, so your second solution of -5 is also correct.

In conclusion, the solutions to $x^2+7x+10=0$ are $\boldsymbol{x=-2}$ and $x=-5$.

This next example involves solving an equation that isn't initially set to equal 0 so requires an additional step in the process.

➦ EXAMPLE Solve the following quadratic equation by factoring:

$$x^2+5x-24=12$$

You know that to use the zero product principle, you need to set the equation equal to 0. To do this, you can subtract 12 from both sides. Now you can determine if you can factor the quadratic expression to use the zero product principle to find the solutions.

$$\begin{aligned} x^2+5x-24 &= 12 \\ -12 &\;\; -12 \\ x^2+5x-36 &= 0 \end{aligned}$$

Next, you need to find two numbers that multiply to -36 and add to 5. The factor pairs of -36 are listed here, and the pair of numbers that also add to 5 are (-4, 9).

Factors of -36	Add to 5
1, -36 ✘	$1+(-36)=-35$
-1, 36 ✘	$(-1)+36=35$
2, -18 ✘	$2+(-18)=-16$
-2, 18 ✘	$(-2)+18=16$
3, -12 ✘	$3+(-12)=-9$
-3, 12 ✘	$(-3)+12=9$
4, -9 ✘	$4+(-9)=-5$
-4, 9 ✔	$(-4)+9=5$
-6, 6	

Now you can factor your expression as:

$$(x-4)(x+9)=0$$

Set each factor equal to 0 to write two separate equations, and solve accordingly, to arrive at your two solutions, x equals 4 and x equals -9.

$$\begin{aligned} x-4 &= 0 & x+9 &= 0 \\ +4 &\;\; +4 & -9 &\;\; -9 \\ x &= 4 & x &= -9 \end{aligned}$$

In conclusion, the solutions to $x^2+5x-24=12$ are $x=4$ and $x=-9$.

TRY IT

Consider the following equation:

$$4x^2+16x=0$$

Solve this equation. You'll notice that each term has common factors that can be factored out. +

Each term has common factors of 4 and x, which can be factored out. Therefore, your equation becomes:

$$4x(x+4)=0$$

Now you can use the zero product principle to set each factor equal to 0 to find the solutions, x equals 0 and x equals -4.

$$\frac{4x}{4}=\frac{0}{4} \qquad\qquad x+4=0$$
$$x=0 \qquad\qquad \begin{aligned} -4 \quad -4 \\ x=-4 \end{aligned}$$

SUMMARY

Today you learned that when quadratic equations are set equal to 0, the solutions are the values for x that make the expression equal to 0. These **solutions to quadratic equations** are commonly referred to as zeroes or roots. You also learned about **the zero product principle**, which states that if a product of two factors is equal to 0, then one of the factors must be 0. Lastly, you learned that quadratic equations are set equal to 0 in order to solve them using different methods, including **solving quadratic equations by factoring**.

Source: This work is adapted from Sophia author Colleen Atakpu.

FORMULAS TO KNOW

Quadratic Equation

$$ax^2+bx+c=0$$