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Solving a Quadratic Equation using Square Roots

by Sophia

WHAT'S COVERED

This tutorial covers how to solve a quadratic equation using square roots, through the discussion of:

1. Square Roots: A Review

In review, the square root of x is a number whose product with itself is x.

EXAMPLE As shown below, 3 times 3 is 9 and -3 times -3 is 9. Therefore, the square root of 9 equals 3 and the square root of 9 also equals -3.

$$3 \cdot 3 = 9 \qquad (-3)(-3) = 9$$
$$\sqrt{9} = 3 \qquad \sqrt{9} = -3$$

DID YOU KNOW

There are always two square roots for any positive number. You can use a plus/minus symbol as a shorthand to indicate the positive and negative solutions, writing the square root of 9, for example, as:

$$\sqrt{9} = \pm 3$$

2. Using Square Roots to Solve Quadratic Equations

Although factoring is a good method for solving a quadratic equation, when there’s no middle term or x term, you can solve the quadratic equation by isolating the variable. Isolating the variable uses inverse operations to undo operations applied to the variable. Inverse operations include:

- Addition/Subtraction
- Multiplication/Division
- Squaring/Taking the Square Root.

↪ EXAMPLE Suppose you want to solve the equation:

$$2x^2 - 8 = 0$$

You can start to isolate the x variable by adding 8 to both sides of the equation.

$$\begin{aligned} 2x^2 - 8 &= 0 \\ +8 \quad &\; +8 \\ 2x^2 &= 8 \end{aligned}$$

Next, you can divide both sides by 2, to further isolate the x variable.

$$\frac{2x^2}{2} = \frac{8}{2}$$
$$x^2 = 4$$

Finally, you can take the square root of both sides to undo the 2 exponent that is squaring the x. This gives you a solution of x equals a positive or negative 2, so this equation has two solutions.

$$\sqrt{x^2} = \sqrt{4}$$
$$x = \pm 2$$

HINT

Remember, taking the square root of a positive number gives two results, one positive and the other negative.

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3. Square Root of an Expression

You can also solve a quadratic equation by taking the square root of an expression.

↪ EXAMPLE Suppose you want to solve the expression:

$$(3x + 3)^2 = 36$$

Notice that there are multiplication and addition operations being applied to the x variable in the parentheses, which means that you undo them last, because, in general, you use the reverse order of operations when solving an equation.

To solve the equation, start by canceling out the squaring operation by taking the square root of both sides. When you do this, on the left side, you have 3x plus 3, because squaring and square root are inverse operations that cancel each other out. On the right side, you have a positive or negative 6.

$$\sqrt{(3x+3)^2} = \sqrt{36}$$
$$3x + 3 = \pm 6$$

This means that you have two results, and solving each will give provide two solutions. To solve the first equation, start by subtracting 3 on both sides, then dividing by 3 on both sides, which provides x equals 1 for your first solution.

$$\begin{aligned} 3x+3&=6 \\ -3\ \ &-3 \\ 3x&=3 \\ \frac{3x}{3}&=\frac{3}{3} \\ x&=1 \end{aligned}$$

To solve the second equation, you also begin by subtracting 3 on both sides, then dividing by 3 on both sides, to give you x equals -3 for your second solution.

$$\begin{aligned} 3x+3&=-6 \\ -3\ \ &-3 \\ 3x&=-9 \\ \frac{3x}{3}&=\frac{-9}{3} \\ x&=-3 \end{aligned}$$

You can verify your solution by substituting both solutions separately back into the original equation. Starting with your first solution, substituting x equals 1 into your equation provides:

$$\begin{aligned} &(3(1)+3)^2=36 \\ &(3+3)^2=36 \\ &6^2=36 \\ &36=36 \end{aligned}$$

Your solution of 1 gives you a true statement, which means it is correct.

Next, check your second solution, x equals -3, by substituting it into the equation.

$$\begin{aligned} &(3(-3)+3)^2=36 \\ &(-9+3)^2=36 \\ &(-6)^2=36 \\ &36=36 \end{aligned}$$

Your solution of -3 also gives you a true statement, so both solutions are correct.

SUMMARY

Today you reviewed **square roots**, noting that the square root of x is a number whose product with itself is x. You also learned that when **solving quadratic equations** with no middle or x term, it is easiest to solve by isolating the variable, using a inverse operation such as taking the square root. Lastly, you learned that taking the **square root of an expression** when solving an equation may lead to having two solutions to the equation.

Source: This work is adapted from Sophia author Colleen Atakpu.

