Solving a Quadratic Equation using the Quadratic Formula

by Sophia

WHAT'S COVERED

This tutorial covers how to solve quadratic equations using the quadratic formula, through the definition and discussion of:

1. Quadratic Equations: A Review

In review, a quadratic equation is an equation that can be written in the following form, where a, b, and c are real numbers:

$$ax^2 + bx + c = 0$$

Factoring, or variable isolation, may be used to solve some quadratic equations, but not all. The quadratic formula, however, can be used to find solutions to *all* quadratic equations, even when factoring or variable isolation is difficult or impossible. Therefore, sometimes it is necessary to use the quadratic formula to find solutions to a quadratic equation.

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2. The Quadratic Formula: A Review

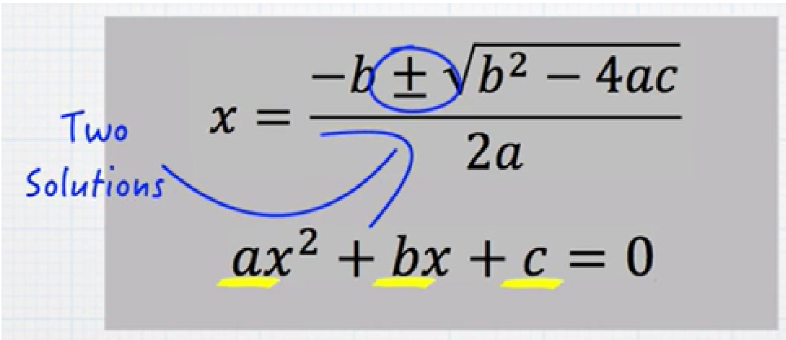
You may recall that **the quadratic formula** states that the solution(s) to a quadratic equation, x, are equal to:

FORMULA TO KNOW

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The values for a, b, and c in the quadratic formula come from the values of a, b, and c in the quadratic equation. The plus-minus symbol here indicates that a quadratic equation may have two solutions.



DID YOU KNOW

When working with the quadratic formula, you often have to simplify square roots using **the product property**, as shown below, or by recognizing perfect squares.

FORMULA TO KNOW

Product Property of Roots

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

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3. Solving Quadratic Equations with the Quadratic Formula

EXAMPLE Suppose you want to solve the quadratic equation:

$$x^2 + 7x - 4 = 0$$

You can solve this equation using the quadratic formula. From the equation, you can see that there is no written number in front of x^2, meaning that there is an implied coefficient of 1. Therefore, a equals 1. You can also see that b equals 7 and c equals -4.

$$1x^2 + 7x - 4 = 0$$

Substituting these values into the formula provides:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-4)}}{2(1)}$$

You can simplify the numerator and the denominator separately. Simplifying the denominator is simple:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-4)}}{2}$$

Next, you can simplify the numerator, which is more complicated, because it involves the plus-minus symbol, square roots, and other operations. You start underneath the square root. moving left to right. 7^2 is 49, and 4 times -1 is -16. You now have 49 minus -16, which is the same as 49 plus 16, which equals 65. The square root of 65 cannot be further simplified, so you'd leave it as written.

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-4)}}{2}$$
$$x = \frac{-7 \pm \sqrt{49 - (-16)}}{2}$$
$$x = \frac{-7 \pm \sqrt{65}}{2}$$

You now have the following solution, which can be separated into its two parts by separating the plus and the minus symbols.

$$x = \frac{-7 \pm \sqrt{65}}{2}$$
$$x = \frac{-7 + \sqrt{65}}{2} \qquad x = \frac{-7 - \sqrt{65}}{2}$$

You can further simplify both of these fractions into two separate fractions each:

$$x = \frac{-7}{2} + \frac{\sqrt{65}}{2} \qquad x = \frac{-7}{2} - \frac{\sqrt{65}}{2}$$

TRY IT

Consider the following quadratic equation:

$$2x^2 - 8x - 3 = 0$$

Use what you've learned so far to solve the following quadratic equation, using the quadratic formula. +

From the equation, you can see that:

- a = 2
- b = -8
- c = -3

Notice that b is *already* a negative number, so when you substitute it into the formula, you have -(-8), which is +8.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-3)}}{2(2)}$$
$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(-3)}}{2(2)}$$

Start by simplifying in the denominator:

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(-3)}}{4}$$

Next, simplify in the numerator, starting with the operations underneath your square root.

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(-3)}}{4}$$
$$x = \frac{8 \pm \sqrt{64 - (-24)}}{4}$$
$$x = \frac{8 \pm \sqrt{88}}{4}$$

Now you can simplify the square root of 88 as the square root of 4 times the square root of 22, using the product property of radicals. The square root of 4 is 2, so your solution is:

$$x = \frac{8 \pm \sqrt{88}}{4}$$
$$x = \frac{8 \pm \sqrt{4} \cdot \sqrt{22}}{4}$$
$$x = \frac{8 \pm 2\sqrt{22}}{4}$$

You can separate this into two solutions:

$$x = \frac{8 + 2\sqrt{22}}{4} \qquad x = \frac{8 - 2\sqrt{22}}{4}$$

You can also separate each of these solutions into separate fractions:

$$x = \frac{8}{4} + \frac{2\sqrt{22}}{4} \qquad x = \frac{8}{4} - \frac{2\sqrt{22}}{4}$$

In this form, you can see that you can simplify these fractions further, so your fully simplified solutions are:

$x = 2 + \frac{\sqrt{22}}{2}$ $\qquad$ $x = 2 - \frac{\sqrt{22}}{2}$

SUMMARY

Today you reviewed **quadratic equations**, noting that factoring, or variable isolation, may be used to solve some, but not all, quadratic equations. You also reviewed **the quadratic formula**, which can be used when **solving all quadratic equations**, even when factoring or variable isolation is difficult or impossible. You learned that before using the quadratic formula, the equation must be equal to 0 in order to determine the correct values of a, b, and c to use in the formula.

Source: This work is adapted from Sophia author Colleen Atakpu.

FORMULAS TO KNOW

Product Property of Square Roots

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$