Solving a System of Linear Equations by Graphing

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to identify the correct graph and intersection point for a system of linear equations. Specifically, this lesson will cover:

1. Solutions to a System of Linear Equations

A solution to a system of linear equations is a specific coordinate (x, y) that satisfies all equations in the system. Graphically, the solution, then, is the intersection point of all lines in the system. The intersection between all lines in the system represents a single (x, y) coordinate that satisfies all equations in the system.

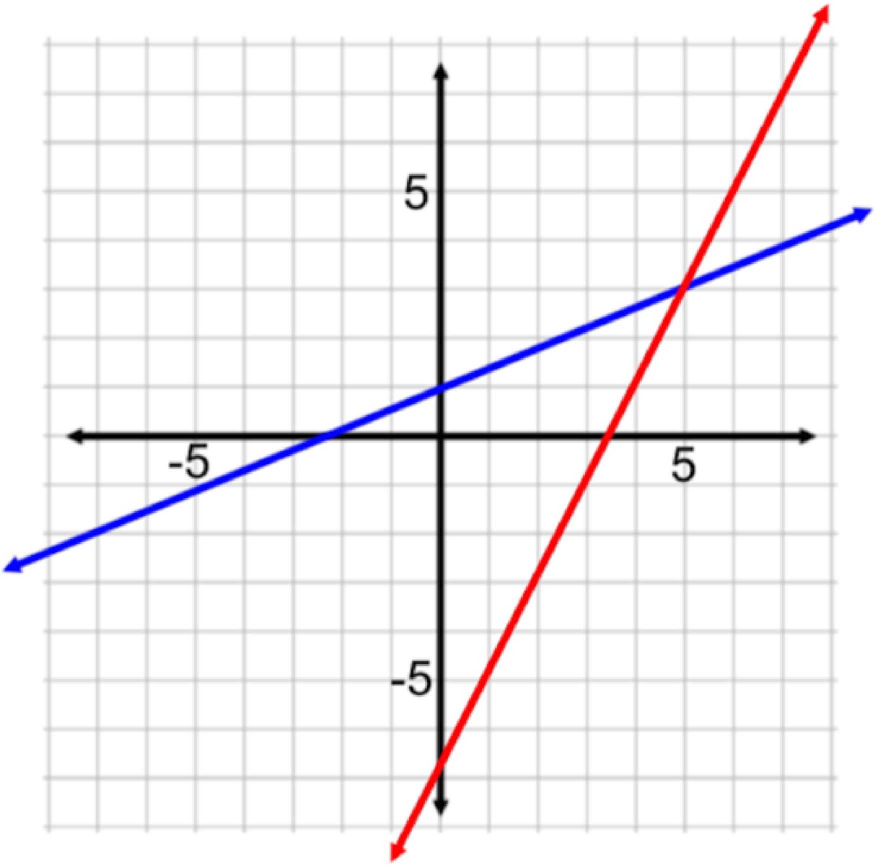
A few cautions about solving with a graph: accuracy is key. If the scale or graph is not easily read, you may end up with only approximate answers, rather than exact solutions to the system.

2. Using a Graph to Solve a System of Equations

For some systems of equations, only a graph is provided, rather than the equations.

The graph below shows two lines that make up a system of equations.

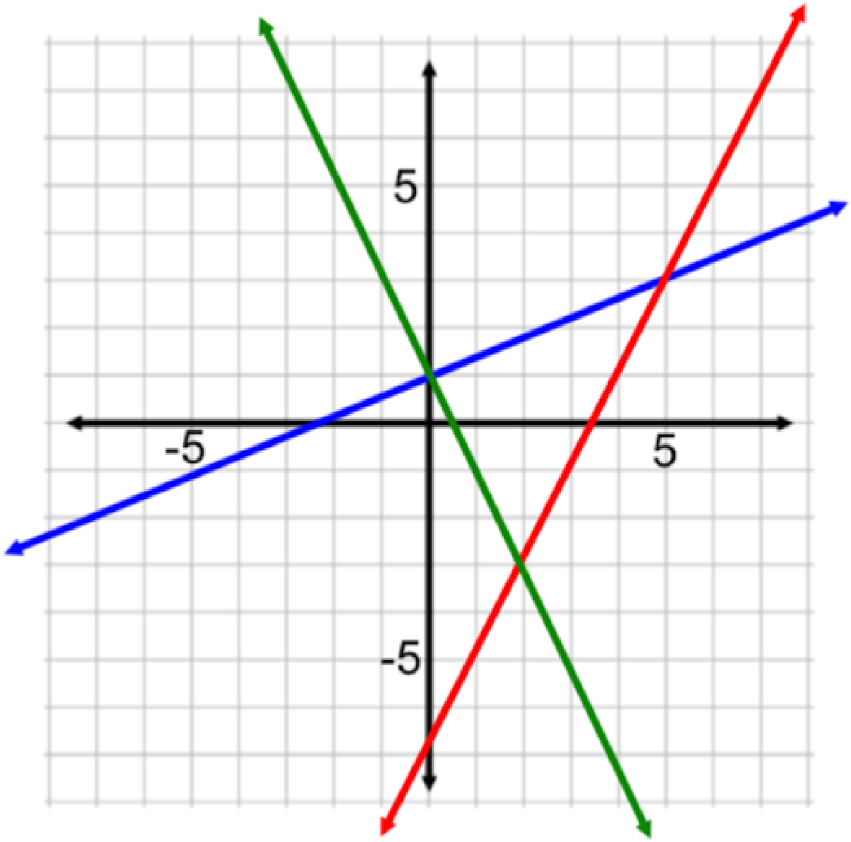
EXAMPLE What is the solution to the following system of equations?



We can see that the blue and red lines intersect at the point (5, 3). This means that (5, 3) is the solution to our system, or in other words, $x = 5$ and $y = 3$ will satisfy both equations in our system.

However, take a look at this next graph which shows three lines that make up a system of equations:

EXAMPLE What is the solution to the following system of equations?



We see a couple of intersection points: (5, 3) is the intersection of the blue and red lines, (2, -3) is the intersection of the red and green lines, and (0, 1) is the intersection of the blue and green lines. Are these all solutions to the system of equations? Actually, none of them are solutions. This system actually has no solutions whatsoever. This is because there is no point of intersection between all three lines that make up the system.

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3. Solving a System of Equations by Graphing

How can you use a graph to solve a given system of equations? You'll first take the equations in the system, graph each equation, and look for the intersection point.

EXAMPLE Solve the following system of equations by graphing:

$$\begin{aligned} 2x - y &= -4 \\ x + y &= 7 \end{aligned}$$

The first thing we need to do is graph the system of equations. Here, our equations are given to us in standard form. Equations in standard form allow us to easily calculate x- and y-intercepts. Remember, the general x-intercept has the coordinate (x, 0) so we can plug in 0 for y and solve for x. For the y-intercept, the general coordinate is (0, y), so we can plug in 0 for x and solve for y.

Let's find the intercepts for our first equation:

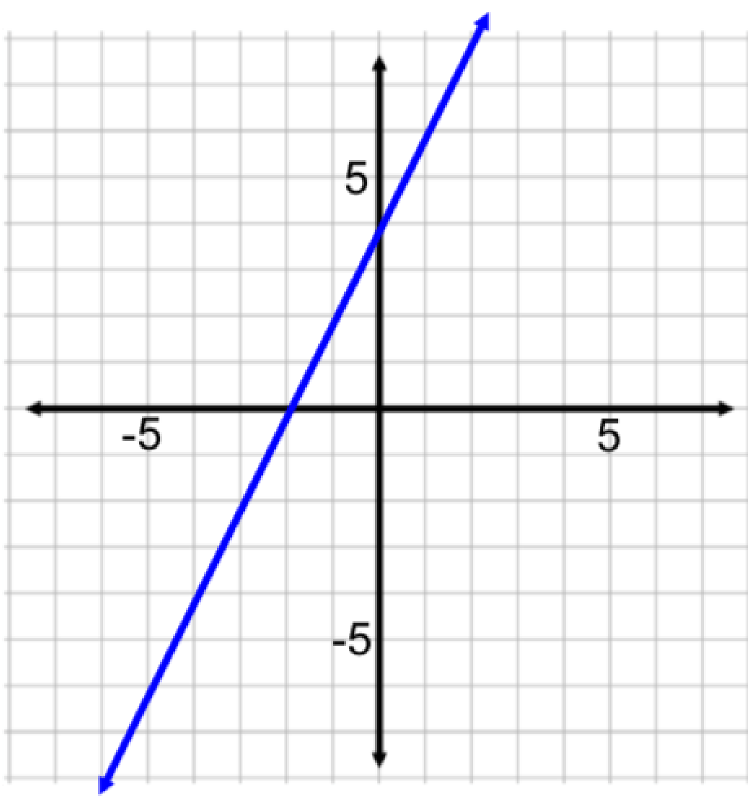
$2x - y = -4$	For the first equation, find the x-intercept by plugging in 0 for *y*
$2x - 0 = -4$	Subtract 0 from 2*x*
$2x = -4$	Divide both sides by 2
$x = -2$	The x-coordinate to the x-intercept

The x-intercept for $2x - y = -4$ is (-2, 0).

$2x - y = -4$	For the first equation, find the y-intercept by plugging in 0 for *x*
$2(0) - y = -4$	Multiply 2 and 0
$-y = -4$	Divide both sides by -1
$y = 4$	The y-coordinate to the y-intercept

The y-intercept for $2x - y = -4$ is (0, 4).

Now we have two points to graph the first equation, (-2, 0) and (0, 4), and can graph the first line to our system.



We follow the same process with the other equation in the system to find the intercepts to graph the line:

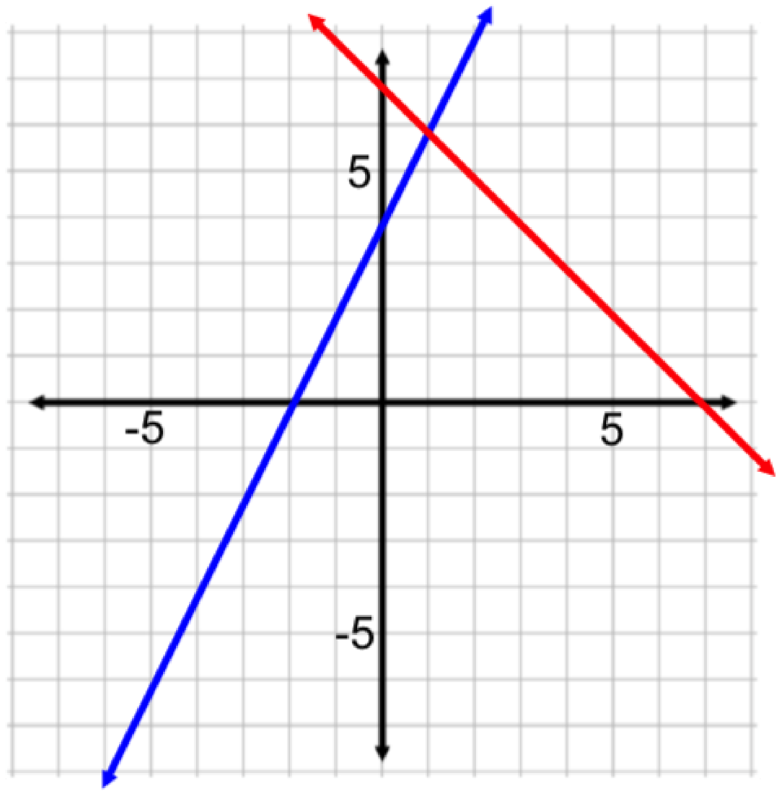
$x+y=7$	For the second equation, find the x-intercept by plugging in 0 for *y*
$x+0=7$	Add 0 and *x*
$x=7$	The x-coordinate to the x-intercept

The x-intercept for $x+y=7$ is (7,0).

$x+y=7$	For the second equation, find the y-intercept by plugging in 0 for *x*
$0+y=7$	Add 0 and *y*
$y=7$	The y-coordinate to the y-intercept

The y-intercept for $x+y=7$ is (0, 7).

Now we have two points for our second equation, (0, 7) and (7, 0), and can graph the second line:



Finally, we see where the two lines intersect. Here, we can see that they intersect at the point (1, 6). This is the solution to our system.

SUMMARY

Solutions to a system of equations is a specific coordinate (*x*, *y*) that satisfies all equations in the system. Graphically, the solution is the intersection point of all lines in the system. We can also **use a graph to find solutions to a system** by finding the only point that satisfies all equations in the system. One of the easier methods is **solving a system of equations by graphing**. Once the lines are graphed, we can identify the point of intersection, which will be the solution.

