Solving a System of Linear Equations using the Addition Method

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WHAT'S COVERED

In this lesson, you will learn how to solve a system of linear equations by using the Addition Method. Specifically, this lesson will cover:

1. Solutions to a System of Equations

A solution to a system of linear equations is a specific coordinate pair (x, y) that satisfies all equations in the system. It is important that the solution satisfies every equation in the system, not just one; otherwise, it doesn't represent a solution to the entire system.

There are several ways to find solutions to a system of linear equations, such as by graphing or using the substitution method. This lesson focuses on one method known as the addition method or the elimination method.

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2. The Addition Method

The main goal of the **addition method** is to add the equations that make up the system so that one of the variable terms will cancel, resulting in an equation with only one variable. From there, we can solve for that variable, and use that value to substitute it back into other equations in the system, and eventually solve for the remaining variable(s). This is why this method is also referred to as the elimination method; because the goal is to eliminate one of the variable terms through equation addition.

In order for a term to cancel when we add equations, we just need to have a variable term in one equation, and its opposite in another equation.

EXAMPLE If we have an $8x$ in one equation and a $-8x$ in another equation, we can add the two equations together, and there will be no x term.

Let's take a look at a concrete example:

EXAMPLE Use the Addition Method to find the solution to the following system of equations:

$$2x - 3y = 9$$
$$5x + 3y = 5$$

As you can see in the system, the variables are lined up so that the x-terms come first, the y-values come second, and the constants are last. We can simply add the lines together:

$2x - 3y = 9$ $5x + 3y = 5$	Add equations together and the y-terms will cancel
$7x = 14$	Divide both sides by 7
$x = 2$	Our solution for *x*

We noticed that we had opposite y-terms in our system. In one equation, we had a positive 3*y*, and in the other, we had a negative 3*y*. When adding the equations together, our x-terms summed to 7*x*, and our constant terms summed to 14, but what happened to the y-terms? They disappeared because the sum of a quantity and its opposite is zero. This eliminated an entire variable from our equation, allowing us to easily solve for *x*.

This doesn't represent our full solution to the system, however. Now that we know that 2 is our x-coordinate, we need to find the associated y-coordinate. To do so, choose any equation in the system; it does not matter which one we choose. We'll substitute 2 in for *x* and solve for *y*. This is shown below:

$2x - 3y = 9$	Using the first equation in the system, plug in 2 for *x*
$2(2) - 3y = 9$	Multiply 2 and 2
$4 - 3y = 9$	Subtract 4 from both sides
$-3y = 5$	Divide both sides by -3
$y = -\frac{5}{3}$	Our solution for *y*

The solution to the system is $(2, -\frac{5}{3})$.

BIG IDEA

Use the addition method when you notice like-terms with opposite coefficients between two equations. When you add the equations, the variable term will disappear from the equation, because the opposite coefficients canceled each other out.

TERM TO KNOW

Addition Method

Also called the elimination method, a strategy to solving a system of equations by adding equations in order to cancel variable terms.

3. Multiplying by -1

Using the Addition Method is ideal when we recognize a variable term in one equation, and its opposite in another equation. Unfortunately, this isn't always the case.

↱ EXAMPLE Find the solution to the following system of equations:

$$3x - 4y = 12$$
$$3x + 8y = 21$$

Adding the two equations as they are would not cancel any variable term. The y-terms definitely do not cancel in their current form. We have a -4 coefficient in front of y in one equation, and +8 coefficient in front of y in the other. -4 and +8 are not opposites; they do not sum to zero.

Look at the x-terms in each equation, however. Wouldn't it be nice if one of them were negative? That way, we would be able to add the two equations and eliminate the x-variable! Let's take one of the equations, and multiply the entire equation by -1:

$3x+8y=21$	Using the second equation in the system, multiply the entire equation by -1
$-1(3x+8y=21)$	Evaluate
$-3x-8y=-21$	An equivalent equation to the original

When we multiplied by -1, every term throughout the entire equation changed sign: positive terms became negative, and negative terms became positive. As a result, we now have two terms that are opposites of each other in our system of equations, and we can use the addition method to eliminate a variable term. This is shown below:

$3x-4y=12$ $-3x-8y=-21$	Using our system of equations, add the two equations together and the x-terms will cancel
$-12y=-9$	Divide both sides by -12
$y=\frac{-9}{-12}$	Change to a decimal
$y=0.75$	Our solution for y

Now that we have a value for y, we can once again use any equation in our system to solve for x through back-substitution:

$3x-4y=12$	Using the first equation, plug in 0.75 for y
$3x-4(0.75)=12$	Multiply 4 and 0.75
$3x-3=12$	Add 3 to both sides
$3x=15$	Divide both sides by 3
$x=5$	Our solution for x

The solution to the system is (5, 0.75).

BIG IDEA

Multiply an equation by -1 when you notice two identical terms between two equations. Multiplying by -1 will turn one of the identical terms into a like-term with opposite coefficients, allowing you to add the equations to cancel a variable term.

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4. Multiplying by a Scalar Value

Sometimes multiplying one of the equations by -1 doesn't help. You may need to multiply it by another factor. Let's consider the following system of equations

EXAMPLE Find the solution to the following system of equations:

$$\begin{aligned} 2x + 3y &= 8 \\ -x - 2y &= -3 \end{aligned}$$

Recall that in the above example, we noted -4*y* and 8*y* would not eliminate *y*, even though one term was positive and one term was negative. Looking at this example, what would make the x-terms eliminate through addition? If we could turn -*x* into -2*x*, or if we could turn 2*x* into *x*, we would be in business. We can do just this by multiplying one of the equations by a scalar value. Let's take our second equation, and multiply it by 2:

$-x - 2y = -3$	Using the second equation in the system, multiply the entire equation by 2
$2(-x - 2y = -3)$	Distribute the 2 into every term
$-2x - 4y = -6$	An equivalent equation to the original

We can use this equivalent equation and add it to the other equation in our system to eliminate the x-terms altogether, allowing us to solve for *y*.

$\begin{aligned} 2x + 3y &= 8 \\ -2x - 4y &= -6 \end{aligned}$	Using our system of equations, add the two equations together and the x-terms will cancel
$-y = 2$	Divide both sides by -1
$y = -2$	Our solution for *y*

With a solution for *y*, we can plug -2 in for y in any of the other equations in the system and find our solution for *x*.

$2x + 3y = 8$	Using the first equation, plug in -2 for *y*
$2x + 3(-2) = 8$	Multiply 3 and -2
$2x - 6 = 8$	Add 6 to both sides
$2x = 14$	Divide both sides by 2
$x = 7$	Our solution for *x*

The solution for the system of equations will be (7, -2).

BIG IDEA

Multiply an equation by a scalar value when you notice that a coefficient of a variable term is a multiple of its like-term in another equation. You can even multiply by negative scalar values in order to create equations with opposite like-terms. Doing so will allow you to add the equations in order to eliminate a variable term.

SUMMARY

The **solutions to a system of equations** is a specific coordinate pair (x, y) that satisfies all equations in the system. **The addition method**, also called the elimination method, involves adding equations together to cancel out or eliminate one of the variables. We may need to multiply one or both of the equations by a number so that when the equations are added, one of the variables will be eliminated. **Multiplying by -1** or **multiplying by a scalar value** may be necessary with the addition method.

TERMS TO KNOW

Addition Method

Also called the elimination method, a strategy to solving a system of equations by adding equations in order to cancel variable terms.