Solving an Exponential Equation

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WHAT'S COVERED

In this lesson, you will learn how to solve an exponential equation by rewriting the base. Specifically, this lesson will cover:

1. Solving an Exponential Equation

When solving for any type of equation, a sound strategy is to apply inverse operations to undo operations being performed to the variable. Doing so isolates the variable to one side of the equation, where we can then evaluate the other side to find our solution.

In general, exponential equations can be solved by applying a logarithm to both sides of the equation. This is because logarithms and exponents are inverse operations. However, we will explore this method later in a different lesson. In this lesson, we focus on how to analyze the bases involved in the equations. If the bases are the same, or if they can be rewritten to match, we can actually solve exponential equations without using logarithms.

2. Exponential Equations with the Same Base

If the bases are the same in exponential equations, we can set the exponents equal to each other, and isolate the variable as we normally do with other equations.

EXAMPLE Solve for *x* in the equation $6^{2x+9} = 6^{-5+2}$.

In both exponential expressions, the base is 6. This means that the quantity of the exponent for 6 is the same on both sides of the equation, therefore $2x+9$ and $-5x+2$ must be equal quantities. We can create an equivalent equation that is actually linear in nature, and solve for x:

$6^{2x+9}=6^{-5x+2}$	Set exponents equal to each other
$2x+9=-5x+2$	Add $5x$ to both sides
$7x+9=2$	Subtract 9 from both sides
$7x=-7$	Divide both sides by 7
$x=-1$	Our solution

3. Rewriting the Base

When we are working with exponential equations in which the base numbers are not the same, it may appear as though we cannot solve using the strategy described in the section above. However, by closely examining the base numbers, we may be able to rewrite one or more of the bases in order to create an equivalent equation with common bases. If we can do this, we can solve the equation using a similar strategy as before.

HINT

When we use this strategy, we will need to apply the Power of a Power Property of Exponents. This property allows us to multiply exponents in cases where a base number is raised to an exponent power and then raised to an exponent power again.

FORMULA TO KNOW

Power of a Power Property of Exponents

$(a^n)^m = a^{nm}$

EXAMPLE Solve for x in the equation $4^{x+3}=8^{x-1}$.

At first glance, it may appear as though we cannot solve this equation using our strategy from before. However, we notice that both 4 and 8 are powers of 2. That is, 4 is the same as 2^2 and 8 is the same as 2^3. Let's make these substitutions in our equation by rewriting each base.

$4^{x+3}=8^{x-1}$	Rewrite with same base of 2
$(2^2)^{x+3}=(2^3)^{x-1}$	Equivalent equation

We can now multiply the two exponents on each side of the equation using the Power of Powers Property of Exponents and then solve as we did in the first example.

$(2^2)^{x+3} = (2^3)^{x-1}$	Use Power of Powers Property and multiply exponents
$2^{2x+6} = 2^{3x-3}$	Set exponents equal to each other
$2x+6 = 3x-3$	Add 3 to both sides
$2x+9 = 3x$	Subtract $2x$ from both sides
$9 = x$	Our solution

When $x = 9$, the two sides of the equation are equal. We can test this by plugging 9 in for x back into the original equation.

$4^{x+3} = 8^{x-1}$	Plug in 9 for x
$4^{9+3} = 8^{9-1}$	Evaluate operations in exponents
$4^{12} = 8^8$	Evaluate result using calculator
$16{,}777{,}216 = 16{,}777{,}216$	This is a true statement

SUMMARY

There are two methods for **solving exponential equations**. For **exponential equations with the same base**, then simply solve by setting the exponents equal to each other. If the bases are not the same, then you will need to **rewrite the base** and use properties of exponents to obtain an equation with common bases. Then, write and solve an equation using just the expressions in the exponents.

FORMULAS TO KNOW

Power of a Power Property of Exponents

$(a^n)^m = a^{nm}$