Solving Equations by Combining Like Terms

by Sophia

WHAT'S COVERED

This tutorial covers solving equations by combining like terms, through the exploration of:

1. Introduction to Solving Equations with Like Terms

Solving both simple and complex equations is a skill that is fundamental to all levels of mathematics. Many equations will require simplification by combining like terms on either side of the equation before using inverse operations to solve the equation. Solving equations is used to solve problems in mathematics as well as science fields, such as physics or chemistry, when balancing chemical equations.

As you may recall, a term is a collection of numbers, variables, and powers combined through multiplication. A coefficient is the number in front of a variable that acts as a factor or multiplier. Like terms are terms with the same variable and variable power, and *only* like terms can be combined with addition or subtraction.

EXAMPLE Suppose you want to simplify the expression: $3x^2+3y^2+12x-2x$.

The only like terms are 12x and negative 2x, and the subtraction sign in front of the 2x is the same as adding a negative 2x. These are the only two terms that can be combined with addition or subtraction, because they are the only two like terms in the expression. You can combine these two like terms by adding their coefficients to get 10x. Notice that the variable and exponent remain unchanged when you combine like terms. Therefore, your simplified expression is:

$3x^2+3y^2+10x$

2. Properties of Equality and Inverse Operations

An equation is a mathematical statement that says two expressions or quantities are equal or have the same value. You may recall that there are several properties of equality that you can use to help solve equations or determine the variable in an equation:

Property of Equality	FORMULA TO KNOW	Description
Addition Property of Equality	$If\ a=b,\ then\ a+c=b+c$	The addition property states that if a equals b, and c is any number, then a plus c is equal to b plus c. Adding c on both sides of the equation still provides a true statement.
Subtraction Property of Equality	$If\ a=b,\ then\ a-c=b-c$	The subtraction property states that if a equals b, and c is any number, then a minus c is equal to b minus c. Subtracting c from both sides of the equation still provides a true statement.
Multiplication Property of Equality	$If\ a=b,\ then\ a\times c=b\times c$	The multiplication property states that if a is equal to b, and c is any number, then a times c is equal to b times c. Multiplying by c on both sides of the equation still provides a true statement.
Division Property of Equality	$If\ a=b,\ then\ a\div c=b\div c$ $(c\ must\ be\ a\ non-zero\ number)$	The division property states that if a is equal to b, and c is any *non-zero number*—you can't divide a number by 0—then a divided by c is equal to b divided by c. Dividing by c on both sides of the equation still provides a true statement.

Remember, these properties state that whatever is done on one side of the equal sign must be done on the other side to maintain an equation, or a true statement.

In addition to the properties of equality, you also use inverse operations when solving equations, to undo operations in an equation in order to isolate the variable or unknown quantity that you want to know. The inverse operations are:

- Addition/Subtraction
- Multiplication/Division
- Squaring/Taking the Square Root

3. Solving Equations by Combining Like Terms

Now that you've reviewed the properties of equality and inverse operations, you can apply them to solve an equation that contains like terms. Solving an equation means isolating the variable on one side of the equal sign with everything else on the other side. You use reverse order of operations to isolate the variable, meaning you use the acronym PEMDAS *backwards*. Therefore, addition and subtraction are undone before any multiplication or division is undone.

EXAMPLE Suppose you want to solve the equation: $3x-8=2x+1$.

$3x-8=2x+1$	Our Equation
$3x-8=2x+1$ $+8 \quad +8$	Start by adding 8 to both sides, which will undo subtracting 8 on the left side.
$3x=2x+9$ $-2x \quad -2x$	Next, we will need to subtract 2x from both sides to undo the 2x on the right side.
$\frac{1x}{1}=\frac{9}{1}$	Now we will divide by 1 to cancel out the coefficient 1 that is being multiplied by the x.
$x=9$	Our Solution

TRY IT

Now that you know how to combine like terms, use your new skills to isolate the variable in the following equation: $-3x+15+x=2x-35+10$.

Solve for x. +

$-3x+15+x=2x-35+10$	Our Equation
$-3x+15+x=2x-35+10$	We will start by combining like terms on the left and on the right. We can combine our x terms on the left and the negative 35 plus 10 on the right
$-2x+15=2x-25$ $-15 \quad -15$	Now, we can start to isolate x by subtracting 15 on both sides of the equation. Subtracting 15 from negative 25 equals negative 40.
$-2x=2x-40$ $-2x \quad -2x$	Next, remove the 2x on the right side of the equation but subtracting 2x from the left and right. This leaves us with negative 4x on the right.
$\frac{-4x}{-4}=\frac{-40}{-4}$	Lastly, divide both sides of the equation by negative 4 to isolate the variable x. This gives us our answer.
$x=10$	Our Solution

SUMMARY

Today you learned about **solving equations with like terms**. You reviewed the definitions of equations, terms, and like terms, as well as the **properties of equality and inverse operations**. You also learned that many equations will require simplification by combining like terms on either side of the equation before using inverse operations to solve, noting that only like terms can be combined with addition or subtraction.

Source: This work is adapted from Sophia author Colleen Atakpu.

FORMULAS TO KNOW

Addition Property of Equality

$If\ a = b, then\ a + c = b + c$

Division Property of Equality

$If\ a = b, then\ a \div c = b \div c$

Multiplication Property of Equality

$If\ a = b, then\ a \times c = b \times c$

Subtraction Property of Equality

$If\ a = b, then\ a - c = b - c$