Solving Equations with Distribution

by Sophia

WHAT'S COVERED

This tutorial covers solving equations with distribution, through the definition and discussion of:

1. The Distributive Property

The distributive property is often used to simplify an equation before solving. It states that a times b plus c is equal to a times b plus a times c. This means that you multiply a by each term inside the parentheses. In other words, the outside factor is “distributed” into the inside factor, which is why it is called the distributive property.

FORMULA TO KNOW

Distributive Property

$$a(b+c)=ab+ac$$

EXAMPLE Use the distributive property to simplify the expression: $3(2+5)$.

$3(2+5)$	Our Expression
$(3\times2)+(3\times5)$	We distribute the three and multiply it by both numbers in the parentheses
$6+15$	3 times 2 is 6 and 3 times 5 is 15.
21	Our Solution

Note that the distributive property is equivalent to simplifying inside the parentheses first and then multiplying by 3. If you add 2 plus 5 in the parentheses, which equals 7, then multiply by 3, you arrive at the same answer: 21. Therefore, both methods provide the same value.

$3(2+5)$	Our Expression

$3(\underline{2+5})$	First, we can add the two numbers in the parentheses. 2 plus 5 equals 7.
$3(7)$	Three times seven equals 21
21	Our Solution

The distributive property has applications in everyday life, including computing total cost, grade point average, and applying taxes to goods and services.

2. The Distributive Property and Negative Numbers

There are several important things to note when distributing a negative number. When you distribute a negative number, the signs of the numbers in the parentheses will switch to the opposite sign.

In the following examples, you will notice that after distributing, the signs of both numbers in the parentheses have changed from positive to negative.

EXAMPLE Evaluate $-4(1+6)$.

$-4(1+6)$	Our Expression
$(-4\times1)+(-4\times6)$	Distribute negative 4 into the parentheses
$-4+-24$	Evaluate
-28	Our Solution

EXAMPLE Evaluate $-(10+5)$.

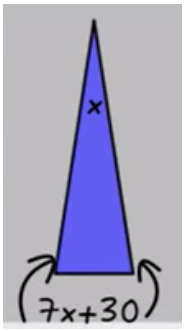
$-(10+5)$	Our Expression
$(-1\times10)+(-1\times5)$	Distribute negative 1 into the parentheses
$-10+-5$	Evaluate
-15	Our Solution

3. Solving an Equation Using the Distributive Property

You may recall that to solve an equation for a variable, you need to isolate the variable on one side of the equation using inverse operations. It's also necessary to combine any like terms on either side of the equation

before using inverse operations to solve. At times, you may also need to use the distributive property in solving an equation.

EXAMPLE Suppose you have an isosceles triangle in which the two congruent angles each measure 30 degrees more than 7 times the third angle, as shown in the picture below. What is the measure of the third angle?



In the problem, the measure of the third angle is the unknown quantity. We will define the variable x as the measure of the third angle.

$2(7x+30)+x=180$

We also know that the other two congruent angles are each 30 degrees more than 7 times the variable x. This means we must multiply 2 (because there are two angles) by 7x plus 30. You then add x, the measure of your third angle. Finally, you know that this expression is going to equal 180 because all angles in a triangle add up to 180 degrees.

$(2\times 7x)+(2\times 30)+x=180$

Next, we will start to solve the equation by using the distributive property. Multiply 2 times 7x and 2 times 30. This will simplify to 14x plus 60. Include the x at the end to complete the expression.

$14x+60+x=180$

Combine the like terms x and 14x to get 15x.

$$\begin{array}{r} 15x+60=180 \\ -60\quad -60 \end{array}$$

Subtract 60 from both sides to isolate the x variable.

$$\frac{15x}{15}=\frac{120}{15}$$

Divide both sides by 15 to get the solution.

$x=8$

Our Solution

You can verify this solution by substituting 8 back into your original equation as x. Simplify using the order of operations, which will provide the true statement 180 equals 180, so your answer is correct.

$$2(7(8)+30)+8=180$$
$$2(56+30)+8=180$$
$$2(86)+8=180$$
$$172+8=180$$
$$180=180$$

IN CONTEXT

Suppose a school club is buying t-shirts for a school fundraiser. The first 10 t-shirts cost $8 each, but the remaining t-shirts are $4 off the original price. How many shirts can they buy if they have $500 to spend?

Let x equal the number of shirts that they can buy. To write your equation, you start by multiplying x by 8, which is the price per t-shirt. They will save $4 for the t-shirts they buy after the first 10 t-shirts, so you subtract 4 multiplied by (x-10). This will equal the total amount they have to spend, which is $500.

Let's use x for the number of t-shirts purchased.

$8x-4(x-10)=500$	Our Equation
$8x+-4(x-10)=500$	On the left side, $8x-4(x-10)$ can be written as $8x+-4(x-10)$ and now we can distribute the -4.
$8x+((-4\times x)-(-4\times 10))=500$	Start by distributing the -4 in front of the parentheses.
$8x+(-4x--40)=500$	-4 times x is -4x. -4 times 10 is -40.
$8x+(-4x+40)=500$	In the parentheses, $-4x--40$ can be rewritten as $-4x+40$.
$8x-4x+40=500$	Once we remove the parentheses, combine like terms. 8x minus 4x equals 4x.
$4x+40=500$ $-40\quad -40$	Subtract 40 from both sides to isolate the x variable.
$\frac{4x}{4}=\frac{460}{4}$	Divide 4 from each side.
$x=115$	Our Solution

Remember to verify that your solution is correct by substituting 115 in for x in your original equation. Using the order of operations, start by simplifying in your parentheses, then move on to multiplication and subtraction. Your final equation is 500 equals 500, so you can see that your solution is correct.

$$8(115)-4(115-10)=500$$
$$8(115)-4(105)=500$$
$$920-420=500$$
$$500=500$$

SUMMARY

Today you learned that when solving an equation, it is necessary to use **the distributive property** and combine any like terms on either side of the equation before using the inverse operations to solve. You also learned that when given an expression with only a negative sign outside of the parentheses, you simplify by **distributing a negative** 1 to the terms inside the parentheses.

Source: This work is adapted from Sophia author Colleen Atakpu.

FORMULAS TO KNOW

Distributive Property

$$a(b+c)=ab+ac$$