Solving Exponential Equations using Logarithms

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to solve an exponential equation by applying logarithmic properties. Specifically, this lesson will cover:

1. Exponential-Logarithmic Relationships

Exponential equations can be equivalently written using a logarithm. In general, we can say that the follow two equations are equivalent:

Exponential Equation	Logarithmic Equation
$y = b^x$	$\log_b(y) = x$

Notice that the base of the exponential expression becomes the base of the logarithmic expression. Also notice that *y*, which was the output of the exponential equation, is the input of the logarithmic operation. This is characteristic of inverse relationships.

When solving exponential equations using logarithms, we often apply two important properties of logarithms: the **power property of logs** and the **change of base property of logs**:

FORMULA TO KNOW

Power Property of Logs
$\log_b(x^n) = n \cdot \log_b(x)$

Change of Base Property of Logs

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

We most often use the change of base property when we wish to use our calculators to evaluate logs. This is because most calculators can only evaluate logs in base 10 (the common log) or e (the natural log).

2. Writing Exponential Equations as Logarithms

EXAMPLE Solve the exponential equation $4^x = 10.556$.

One strategy for solving this equation is to see if 10.556 is a power of 4. If so, it is relatively easy to solve for *x* mentally, as *x* will be an integer, such as 1, 2, 3, and so on. However, there is no integer exponent we can apply to 4 to get a value of 10.556. In this case, it is helpful to write this into an equivalent logarithmic equation:

$4^x = 10.556$	Rewrite using a logarithmic eqution
$x = \log_4(10.556)$	Equivalent equation to $4^x = 10.556$

Now that we have a logarithmic expression for *x*, we can use the change of base property to evaluate the log using our calculator:

$x = \log_4(10.556)$	Apply the Change of Base Property of Logs
$x = \frac{\log(10.556)}{\log(4)}$	Use calculator to evaluate
$x = 1.7$	Our solution

3. Applying Logarithms to Both Sides of an Equation

As with all equations, we can apply inverse operations to both sides of the equation in order to isolate the variable. The inverse operation of an exponent is the logarithm. In this next example, we will see how we can apply the logarithm and other inverse operations to isolate the variable.

EXAMPLE Solve the exponential equation $2(5.5)^x = 168$.

$2(5.5)^x = 168$	Divide by 2 to have only 5.5^x on the left side
$5.5^x = 84$	Apply log of both sides

$\log(5.5^x) = \log(84)$	Apply the Power Property of Logs
$x \cdot \log(5.5) = \log(84)$	Divide both sides by $\log(5.5)$
$x = \dfrac{\log(84)}{\log(5.5)}$	Use calculator to evaluate
$x = 2.6$	Our solution

BIG IDEA

When applying the log to both sides of an exponential equation, this enables us to apply the power property of logarithms. We can move the variable outside of the logarithm, as a scalar multiplier to the log. Then, we are able to isolate the variable by dividing everything else out.

SUMMARY

Recall that since there is an **exponential-logarithmic relationship**, we can **write exponential equations as logarithms** by using properties of logarithms. There is the power property, the change of base formula, and the conversion between logarithmic form and exponential form.

One method of solving exponential equations involves converting the equation from exponential to logarithmic form, and then using the change of base formula to solve. A second method of solving exponential equations involve **applying logarithms to both sides of the equation**, and using the power property of logs to simplify and solve.

FORMULAS TO KNOW

Change of Base Property of Logs

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Power Property of Logs

$$\log_b(x^n) = n \cdot \log_b(x)$$