Solving Logarithmic Equations using Exponents

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to solve a logarithmic equation by rewriting it into an exponential equation. Specifically, this lesson will cover:

1. Exponential-Logarithmic Relationships

Logarithmic equations can be equivalently written using exponents. In general, we can say that the following two equations are equivalent:

Exponential Equation	Logarithmic Equation
$y = b^x$	$\log_b(y) = x$

Notice that the base of the logarithm is the base of the exponential expression. Additionally, the input of the logarithmic function is the output of the exponential function.

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2. Rewriting Logarithmic Equations as Exponential Equations

For some logarithmic equations, it may be helpful to rewrite the equation as an equivalent exponential equation to solve.

EXAMPLE Solve the logarithmic equation $\log_7(x) = 2.8$.

$\log_7(x) = 2.8$	Rewrite into an exponential equation
$7^{2.8} = x$	Evaluate using calculator
$232.42 = x$	Our solution

BIG IDEA

One strategy in solving logarithmic equations is to rewrite it as an exponential equation. In many cases, by doing so, the equation will have an exponential expression on one side of the equation, and an isolated variable on the other side. We can then evaluate the exponential expression to find the solution to the equation.

EXAMPLE Solve the logarithmic equation $4\log_5(x) = 7.6$.

$4\log_5(x) = 7.6$	Divide both sides by 4 to have only $\log_5(x)$ on left side
$\log_5(x) = 1.9$	Rewrite into an exponential equation
$5^{1.9} = x$	Evaluate using calculator
21.28	Our solution

HINT

It is important to the log expression by itself on one side. In the example above, we had to divide both sides by 4 before we could convert from a logarithmic equation to an exponential equation.

3. Using the Log Expression as an Exponent

Another method to solving log equations involves applying the inverse relationship between exponents and logs in a slightly different way than you may be used to. We can use the base of the logarithm as a base of an exponent and place the logarithmic expression as an exponent in the equation. We'll have to do this to both sides of the equation.

EXAMPLE Solve the logarithmic equation $\log_3(6x+9) = 4$.

$\log_3(6x+9) = 4$	Apply the base of the log, 3, as a base of an exponent on both sides.
$3^{\log_3(6x+9)} = 3^4$	On the left side, the argument of the log $6x+9$ remains; on the right side, evaluate 3^4
$6x+9 = 81$	Subtract 9 from both sides
$6x = 72$	Divide both sides by 6
$x = 12$	Our solution

BIG IDEA

This is essentially a more explicit explanation of the relationship between logarithmic equations and exponential equations. We can use the base of the log to create an exponential equation with the same base. Since logarithms and exponents are inverse operations, this undoes any log or exponent operation, leaving only the argument of the log on one side of the equation, and an exponential expression on the other.

SUMMARY

The **exponential-logarithmic relationship** says that we can use properties of logarithms to solve logarithmic equations, such as the power property, change of base formula, and the conversion between logarithmic form and exponential form. One method of solving logarithmic equations involves **rewriting the logarithmic equation to an exponential equation**. A second method involves **using the log expression as an exponent**, which cancels out the logarithm operation, and solving the resulting equation.