Solving Mixture Problems using a System of Equations

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WHAT'S COVERED

In this lesson, you will learn how to predict the correct system of equations for a given situation involving mixtures. Specifically, this lesson will cover:

1. What are Mixture Problems?

Mixture problems involve combining two or more things, such as chemical solutions. Mixture problems can also involve prices, percents, and other concentrations. These types of problems can be solved by setting up and solving a system of equations. Typically, one equation represents the relationship between the concentration amounts given by the problem, and another equation relates the total quantities involved.

For mixture problems, we usually have to do two things:

- define variables
- identify the system of equations

In most cases, we can define variables generically as:

$$x = \text{amount of one item}$$
$$y = \text{amount of another item}$$

The system of equations is made up of two equations:

- $x + y = t$, where t is the total amount
- $Ax + By = Ct$, where A is the price, percent, or concentration of x, B is the price, percent, or concentration of y, and C is the price, percent, or concentration of the total amount, t

Let's take a look at a real-world example.

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2. Setting Up a System of Equations for a Mixture Problem

Let's consider a classic mixture problem. These problems usually involve mixing chemicals with different percent concentrations, in order to yield a specific amount of a specific percent concentration needed for a lab experiment.

↪ EXAMPLE You need to prepare 50 mL of a 17% solution of HCl for a lab experiment. You only have two solutions of HCl available to you: a 10% solution, and a 40% solution. How much of each solution should you combine to yield 50 mL of 17% HCl?

First, we need to define our variables:

$$x = \text{mL of } 10\% \text{ solution}$$
$$y = \text{ mL } 40\% \text{ solution}$$

One equation in our system will represent the relationship between the quantity and the concentration. We can create one part of the equation with the expression $0.1x+0.4y$. Here, we multiplied each quantity, *x* and *y*, by their respective percent concentrations (expressing the percents as decimals: $10\% = 0.10 = 0.1$ and $40\% = 0.40 = 0.4$). When we add these two together, we want to get 50 mL of 17% solution. We can express this as 0.17(50). Putting this together, one equation in our system is:

$$0.1x + 0.4y = 0.17(50)$$
$$0.1x + 0.4y = 8.5$$

The second equation in our system needs to relate the quantities. We need to mix a certain amount of 10% solution with a certain amount of 40% solution to get 50 mL total. This can be expressed using the equation:

$$x + y = 50$$

Now we have a system of equations:

$$0.1x + 0.4y = 8.5$$
$$x + y = 50$$

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3. Solving a System of Equations for a Mixture Problem

There are several ways we can solve a system of equations. We can use the addition method, substitution method, or even solve by graphing. If you are dealing with decimal numbers in your system, it probably isn't the easiest to solve by graphing. You may also have trouble solving by the addition method if you don't have terms that would easily cancel through addition. If those fail, you can try solving by substitution.

↪ EXAMPLE Solve the system of equations to find how many milliliters of 40% and 10% solution we need to mix in order to yield 50 mL of a 17% solution.

$$0.1x + 0.4y = 8.5$$
$$x + y = 50$$

To solve using the substitution method, we take one of our equations and choose a variable to isolate. The second equation in our system is ideal for this because there are no coefficients in front of x or y. It doesn't matter which variable we choose to isolate. Let's pick x:

$x + y = 50$	Using the second equation, isolate x by subtracting both sides by y
$x = 50 - y$	An equivalent equation to the original

Now that we have an expression for x, we can substitute it into the other equation that we didn't use to isolate a variable. To do this, we will re-write the equation, but write $50 - y$ instead of x:

$0.1x + 0.4y = 8.5$	Using the first equation, substitute $50 - y$ in for x
$0.1(50 - y) + 0.4y = 8.5$	Single-variable equation with only y's

What we are left with is a single-variable equation. We can solve this equation and get a value for y, and then use that value of y to solve for x.

$0.1(50 - y) + 0.4y = 8.5$	Using this new equation, solve for y by distributing 0.1 into $(50 - y)$
$5 - 0.1y + 0.4y = 8.5$	Combine like terms
$5 + 0.3y = 8.5$	Subtract 5 from both sides
$0.3y = 3.5$	Divide both sides by 0.3
$y = 11.67$	Our solution for y (rounded)

So far, we know that we'll need to mix 11.67 mL of the 40% solution to yield our desired mixture. To solve for x, we can use any of the other equations in our system (or equivalent equations we have developed), using a fixed value for y, 11.67. Since we want to solve for x, let's use the equation we created that already has x isolated on one side of the equation.

$x = 50 - y$	Using the Equivalent equation for $x + y = 50$, substitute 11.67 in for y
$x = 50 - 11.67$	Subtract 11.67 from 50
$x = 38.33$	Our solution for x

This tells us that 38.33 mL of 10% solution and 11.67 mL of 40% solution will yield 50 mL of 17% solution.

SUMMARY

When determining **what are mixture problems**, a mixture problem involves combining two or more things and then calculating a quantity from the mixture, such as price, percent, or concentration. **Setting up a system of equations for a mixture problem** involves creating equations to represent relationships between known and unknown quantities. When **solving a system of equation for a mixture problem**, use the addition method, substitution method, or even solve by graphing.