# Solving Mixture Problems using Weighted Average

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WHAT'S COVERED

In this lesson, you will learn to use weighted average to calculate the concentration of a mixed solution. Specifically, this lesson will cover:



## 1. Average/Mean

Before we begin discussing weighted average, it may be helpful to review how to calculate the simple average or mean of a data set. In order to find the average, we find the sum of all data values, and then divide by the number of data values in our set.

FORMULA TO KNOW

**Mean/Average**

$$\text{mean} = \frac{\text{sum of terms}}{\text{number of terms}}$$

EXAMPLE Find the average of the set $\{4, 4, 8, 12, 14\}$.

| | |
|---|---|
| $\{4, 4, 8, 12, 14\}$ | Add terms and divide by the number of terms, 5 |
| $\text{average} = \frac{4+4+8+12+14}{5}$ | Evaluate addition in numerator |
| $\text{average} = \frac{42}{5}$ | Divide |
| $\text{average} = 8.4$ | Our Solution |

# 2. Weighted Average

Weighted average is different from simple average in that certain data values carry more weight, or influence the average more heavily. With simple average, every data point is equally represented in the calculation for the mean, but in weighted averages, some data points are multiplied by a number in its calculation. We use this formula for weighted average:

FORMULA TO KNOW

**Weighted Average**

$$\text{weighted average} = \frac{\text{sum of (value} \cdot \text{weight) of each item}}{\text{sum of weights}}$$

Often times, we see weighted average in class grades.

EXAMPLE Suppose you are taking a science course this semester. Tests and projects are worth 50% of your grade, daily assignments are worth 30% of your grade, and participation is worth 20% of your grade. You scored the following on these assessments:

| Assessment | Score | Weight |
|---|---|---|
| Tests and Projects | 83% | 50% |
| Daily Assignments | 94% | 30% |
| Participation | 80% | 20% |

To calculate your final grade for the class, we multiply these assessment scores by their weights:

- Multiply your test/project score 83% by its weight of 50%
- Multiply your assignment score 94% by its weight of 30%
- Multiply your participation grade 80% by its weight of 20%

We'll add these values, then divide by the sum of all the weights, to find your weighted grade for the course. This is illustrated below:

| | |
|---|---|
| $\text{weighted average} = \frac{\text{sum of (value} \cdot \text{weight) of each item}}{\text{sum of weights}}$ | Multiply 83% by 50%, 94% by 30%, 80% by 20% and divide by the sum of the weights |
| $\text{weighted average} = \frac{(0.83)(0.50)+(0.94)(0.30)+(0.80)(0.20)}{0.50+0.30+0.20}$ | Evaluate multiplication in numerator |
| $\text{weighted average} = \frac{0.415+0.282+0.16}{0.5+0.3+0.2}$ | Simplify numerator and denominator |
| $\frac{0.857}{1} = x$ | Divide 0.857 by 1 |
| $0.857 = x$ | Our Solution |

Your overall weighted score in your science course is 85.7%.

HINT

In general, data values in the denominator of the fraction are multiplied by its corresponding weight. The corresponding weights are then added together to form the denominator of the fraction. In this case, the weights summed to 100%, or 1, because the weights of tests/projects, assignments, and participation need to reflect 100% of your grade in total.

# 3. Mixture Problems

Next, we will use the concept of weighted average to set up and solve a mixture problem. We'll use this adapted formula as we work through these types of problems:

FORMULA TO KNOW

**Weighted Average for Mixture Problems**

$$\frac{(C_1)(Q_1)+(C_2)(Q_2)}{Q_1+Q_2}=C_3$$

In this formula,

- $C_1$ is the concentration of Item 1
- $Q_1$ is the quantity of Item 1
- $C_2$ is the concentration of Item 2
- $Q_2$ is the quantity of Item 2
- $C_3$ is the concentration of the combined items

HINT

If you notice, there is no $Q_3$ defined. $Q_3$ would be the quantity of al the combined items. We technically do see this in our formula in the denominator, $Q_1+Q_2$.

Consider the following example:

EXAMPLE To prepare for a lab experiment, you mix two concentrations of HCl (hydrochloric acid): 30 mL of 15% HCl solution, and 20 mL of 40% HCl solution. What is the concentration of the mixed solution?

To solve this problem, we need to multiply the quantity of each solution by its concentration (or its weight to be included in our calculation). This will represent the numerator of our fraction for weighted average. As for the denominator, we divide by the total quantity of the solution (so we add the individual amounts to get a total amount of liquid). The solution is worked out below:

| | |
|---|---|
| $\frac{(C_1)(Q_1)+(C_2)(Q_2)}{Q_1+Q_2}=C_3$ | Plug in the concentrations and quantities for the two known solutions: $C_1=0.15$, $Q_1=30$, $C_2=0.40$, $Q_2=20$ |
| $\frac{(30)(0.15)+(20)(0.40)}{30+20}=C_3$ | Multiply the quantity by its weight in numerator. |
| $\frac{4.5+8}{30+20}=C_3$ | Simplify the numerator and denominator |
| $\frac{12.5}{50}=C_3$ | Divide |
| $0.25=C_3$ | Our Solution |

The concentration of the mixed solution will be 25%.

Some mixture problems will ask us to find specific amounts of each solution that must be mixed together in order to yield a certain amount of a specific concentration. Let's again use the HCl example.

EXAMPLE We have two kinds of solutions already made: a 20% solution, and a 50% solution. To prepare for a lab experiment, we are going to need 120 mL of a 30% solution. How many mL of each solution must be mixed in order to create this mixture?

Let's define variables for this situation:

- $x = mL\ of\ 20\%\ solution$
- $y = mL\ of\ 50\%\ solution$

We have an immediate relationship between *x* and *y*, that $x+y=120$. How does this affect our equation for weighted average?

| | |
|---|---|
| $\frac{(C_1)(Q_1)+(C_2)(Q_2)}{Q_1+Q_2}=C_3$ | Plug in the concentrations for the known solutions: $C_1=0.2$, $C_2=0.5$, $C_3=0.3$ |
| $\frac{0.20x+0.50y}{x+y}=0.30$ | Substitute $x+y=120$ |
| $\frac{0.20x+0.50y}{120}=0.30$ | Multiply both sides by 120 |
| $0.20x+0.50y=36$ | Equivalent weighted average equation |

We have simplified our equation, but it still has two variables. If this equation could be expressed using only one variable, we could solve for that variable, and then use substitution again to solve for the other variable. Let's return to the relationship $x+y=120$. We can write this equivalently as $y=120-x$.

Now that we have an expression for y, we can write $120-x$ into the original equation instead of *y*. The result will be a single-variable equation, and we'll be able to solve for *x*.

| | |
|---|---|
| $0.20x+0.50y=36$ | Substitute $y=120-x$ |

$$0.20x+0.50(120-x)=36$$ Distribute 0.50 into $(120-x)$

$$0.20x+60-0.50x=36$$ Combine like terms

$$-0.30x+60=36$$ Subtract 60 from both sides

$$-0.30x=-24$$ Divide both sides by -0.30

$$x=80$$ Our solution x, the quantity of 20% solution

This tell us that we must use 80 mL of the 20% solution in our mixture. Since we know we need a total of 120 mL, we can deduce that we'll need 40 mL of the 50% solution. This combination will yield 120 mL of 30% solution.

## SUMMARY

Recall that the **average/mean** is finding the sum of all data values, and then divide by the number of data values in our set. A **weighted average** gives you the average of a set of values that may carry different weights. The more weight that a value has, the more it is accounted for in the calculation. We can use the concept of weighted average to calculate **mixture problems**.

## FORMULAS TO KNOW

**Mean/Average**

$$\text{mean}=\frac{\text{sum of terms}}{\text{number of terms}}$$

**Weighted Average**

$$\text{weighted average}=\frac{\text{sum of (value}\cdot\text{weight) of each item}}{\text{sum of weights}}$$

**Weighted Average for Mixture Problems**

$$\frac{(C_1)(Q_1)+(C_2)(Q_2)}{Q_1+Q_2}=C_3$$