Solving Multi-Step Equations

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WHAT'S COVERED

In this lesson, you will learn how to solve multi-step equations involving roots. Specifically, this lesson will cover:

1. Solving an Equation

When solving an equation, our main goal is to isolate the variable we wish to solve for onto one side of the equation, and then evaluate the expression on the other side of the equation. To do this, we use inverse operations to undo the operations on our variable. With single step equations, only one operation is applied to our variable, so only one inverse operation must be applied. Let's review these inverse operations.

Operation	Inverse Operation
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication
Powers	Roots
Roots	Powers

What do we do when we have several operations being performed on the variable we wish to solve for? We know that we must perform inverse operations to both sides of the equation (due to the Rule of Equality for equations), but in what order? Think about the order of operations. These operations are applied to the variable following the order of operations. To undo this, we apply inverse operations in the reverse order of operations

BIG IDEA

To solve multi-step equations, apply inverse operations to both sides of the equation, following the reverse order of operations.

2. An Equation with Distribution

Generally, there are two options when you have an equation involving distribution.

- Option 1: Divide by the outside factor, eliminating the need to distribute anything.
- Option 2: Simplify the expression by distributing the factor.

We will see how to solve an equation both ways:

↱ EXAMPLE Solve the equation $9 = 3(2x - 5)$ using both options.

Option 1: Divide by outside factor

$9 = 3(2x - 5)$	Divide both sides of the equation by 3
$3 = 2x - 5$	Undo -5 by adding 5 to both sides of the equation
$8 = 2x$	Undo multiplication of 2 by dividing both sides by 2
$4 = x$	Our Solution

Option 2: Distribute the factor

$9 = 3(2x - 5)$	Distribute 3 on the right side
$9 = 6x - 15$	Undo subtracting 15 by adding 15 to both sides
$24 = 6x$	Undo multiplication of 6 by dividing both sides by 6
$4 = x$	Our Solution

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3. A Variable in the Denominator

When a variable appears in the denominator of a fraction, it can be difficult to isolate that variable until it is moved into a numerator. When the denominator contains the variable, the first step we take is to multiply the entire equation by the expression in the denominator. This eliminates the variable from the denominator on one side of the equation, and makes it part of the numerator on the other side of the equation.

↱ EXAMPLE

$8 = \frac{48}{2x}$	Multiply both sides of the equation by $2x$
$8(2x) = \frac{48}{2x}(2x)$	$2x$'s on the right side will cancel, leaving 48.
$8(2x) = 48$	Multiply 8 and $2x$
$16x = 48$	Divide both sides by 16
$x = 3$	Our Solution

HINT

Notice that when clearing the variable in the denominator, we multiply the equation by the entire denominator, not just by the variable alone.

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4. An Equation involving a Power or a Root

So far we have seen how to apply inverse operations to undo addition, subtraction, multiplication and division. Lastly, let's see how to apply power and roots to solve a multi-step equation.

↪ EXAMPLE

$3 = \sqrt[3]{12x+3}$	Undo the cube root by cubing both sides
$3^3 = \left(\sqrt[3]{12x+3}\right)^3$	Cubing a cubed root will leave only $12x+3$ on the right side; Evaluate 3^3
$27 = 12x+3$	Undo the addition of 3 by subtracting 3 from both sides
$24 = 12x$	Undo the multiplication of 12 by dividing both sides by 12
$2 = x$	Our Solution

As we can see with the example above, we had an expression underneath a cube root. To undo this, we cubed both sides of the equation. We were left with the expression alone, without a radical, and 3 cubed on the other side. From there, we were able to solve for x by applying our familiar inverse operations.

SUMMARY

The process for **solving an equation** involves using inverse operations in the reverse order of operations or PEMDAS backwards. If possible, it is best to simplify the equation before using those inverse operations. If **a variable is in the denominator** of a fraction, you need to multiply both sides of the equation by that variable to solve the equation. Finally, if solving **an equation with a power or root**, you need to isolate the radical and use a power to cancel it out before doing anything underneath the radical.