

# Solving Problems Involving Percents

by Sophia



## WHAT'S COVERED

In this lesson, you will learn how to calculate the original price of an item that had either a sales tax or discount applied. Specifically, this lesson will cover:

1. Percent Tax
2. Percent Discount

## 1. Percent Tax

Many items that we purchase every day have tax applied to them. Tax represents an increase in the price we must pay for these items, so it's intuitive to think of the original price to be multiplied by a number larger than 1. Tax is also most often a percentage rate, such as 7%. When adding 7% of a quantity to itself, you are really doing two things:

1. Calculating 7% of that item
2. Adding it to 100% of the initial amount

For this reason, we use the factor  $(1 + r)$  to calculate the price of an item with tax included. It is important to note that  $r$  is the tax rate expressed as a decimal, and the 1 represents 100% of the original price.

To calculate the price of an item with tax included, multiply the original price of the item by  $(1 + r)$ , where  $r$  is the percent tax expressed as a decimal.



### FORMULA TO KNOW

#### Tax Formula

$$\text{price with tax} = (1 + r) \cdot (\text{original price})$$

Let's apply this concept to a few basic examples.

↪ **EXAMPLE** A pair of pants costs \$18. There is a 6% tax applied to the cost of the pants. How much is due at checkout for the pants?

$$\text{price with tax} = (1 + r) \cdot (\text{original price})$$

Substitute the known values: original price = 18,  $r = 6\%$ . Note that 6% can be expressed as 0.06

$$\text{price with tax} = 18(1 + 0.06)$$

Add 1 and 0.06

$$\text{price with tax} = 18(1.06)$$

Multiply 18 and 1.06

$$\text{price with tax} = 19.08$$

Our Solution

\$19.08 is due at checkout for the pants with tax.

Percent tax problems can get complicated. Consider the next scenario.

⇒ **EXAMPLE** You manage an art store and are preparing an order for canvas paper from your distributor. You purchase 40 boxes of canvas paper, and the total comes to \$998.20, which includes an 8.5% tax. What is the cost of each box of canvas paper before taxes?

Here, the figure of \$998.20 already includes the 8.5% tax, so as we solve our problem, we have to keep in mind that some dollar amount has increased by 8.5% to cost \$998.2. That dollar amount represents the cost of 40 boxes of canvas paper before tax was applied. A final step will be to divide the cost by the number of boxes to represent the cost of each box before taxes. Our solution is worked out below:

$$\text{price with tax} = (1 + r) \cdot (\text{original price})$$

Substitute the known values: price with tax = 998.20,  $r = 8.5\%$ . There are also 40 boxes so we'll want to multiply the original price we are finding by 40 to find the true cost

$$998.20 = (1 + 0.085)(40x)$$

Add 1 and 0.085

$$998.20 = (1.085)(40x)$$

Multiply 1.085 and 40x

$$998.20 = 43.4x$$

Divide both sides by 43.4

$$23 = x$$

Our Solution

The cost of each box of canvas paper before taxes is \$23.00.

## 2. Percent Discount

Just as percent tax represents an increase in cost, percent discount represents a decrease in cost. Recall that we multiplied the cost of an item by  $(1 + r)$  to represent tax being applied. Similarly, we can multiply the cost of an item by  $(1 - r)$ , where  $r$  is the percent discount.



### FORMULA TO KNOW

#### Discount Formula

$$\text{price with discount} = (1 - r) \cdot (\text{original price})$$

One way to think about  $(1 - r)$  is that we are retaining a certain percentage of the cost after a discount, and that percentage is 100% of the cost minus the percent discount.

⇒ **EXAMPLE** If the percent discount is 30%, we must still pay for 70% of the item.

Let's take a look at a basic percent discount problem, and then we will tackle a more complicated one:

⇒ **EXAMPLE** Last week, a patio umbrella was priced at \$149.99. This week, the item is on sale with a 20% discount. How much does the patio umbrella cost this week?

To solve this problem, we first must represent 20% as a decimal, and subtract it from 1. This is the  $(1 - r)$  factor that will be multiplied by the original price to give us the price after the discount. The solution is worked out below:

price with discount = $(1 - r) \cdot (\text{original price})$	Substitute the known values: original price = 149.99, $r = 20\%$ . Note that 20% is expressed as 0.20, or even just 0.2
price with discount = $(1 - 0.2)(149.99)$	Subtract 0.2 from 1
price with discount = $(0.8)(149.99)$	Multiply 149.99 and 0.8
price with discount = 119.99	Our Solution

The discounted patio umbrella will cost \$119.99 this week.

Our final example is more complicated, because it includes both sales tax and a percent discount.

⇒ **EXAMPLE** You are placing an online order to purchase 8 novels for a book club meeting. The total amount due is \$89.88, which includes a 25% discount on the books, and a 7% sales tax. What is the price of each book, before any discounts or taxes are applied?

In this situation, we are dealing with two rates. The first rate is the percent discount of 25%. As a factor in our equation, we will use  $r_d = (1 - 0.25)$  to represent the percent discount. The second rate is the sales tax of 7%. The factor for this will be  $r_t = (1 + 0.07)$ . The solution to our problem is worked out below:

price with discount AND tax = $(1 - r_d) \cdot (1 + r_t) \cdot \text{original price}$	Substitute the known values: price with discount AND tax = 89.88, $r_d = 25\%$ , $r_t = 7\%$ . There are also 8 novels so we'll want to multiply the original price we are finding by 8 to find the true cost
$89.88 = (1 - 0.25) \cdot (1 + 0.07) \cdot (8 \cdot \text{original price})$	Simplify the addition and subtraction in the parentheses
$89.88 = (0.75) \cdot (1.07) \cdot (8 \cdot \text{original price})$	Multiply 0.75 and 1.07
$89.88 = (0.8025) \cdot (8 \cdot \text{original price})$	Multiply 0.8025 and 8
$89.88 = 6.42 \cdot \text{original price}$	Divide both sides by 6.42
$14 = \text{original price}$	Our Solution

The price of each book is \$14.



## SUMMARY

Percentages can be used for solving everyday problems such as those involving tax or discounts.

**Percent tax** represents an increase in cost, **percent discount** represents a decrease in cost. When calculating the percentages, you have to convert the percentage to decimal form by dividing by 100.

Source: ADAPTED FROM "BEGINNING AND INTERMEDIATE ALGEBRA" BY TYLER WALLACE, AN OPEN SOURCE TEXTBOOK AVAILABLE AT [www.wallace.ccfaculty.org/book/book.html](http://www.wallace.ccfaculty.org/book/book.html). License: Creative Commons Attribution 3.0 Unported License



## FORMULAS TO KNOW

### Discount Formula

$$\text{price with discount} = (1 - r) \cdot (\text{original price})$$

### Tax Formula

$$\text{price with tax} = (1 + r) \cdot (\text{original price})$$