Solving Quadratic Equations

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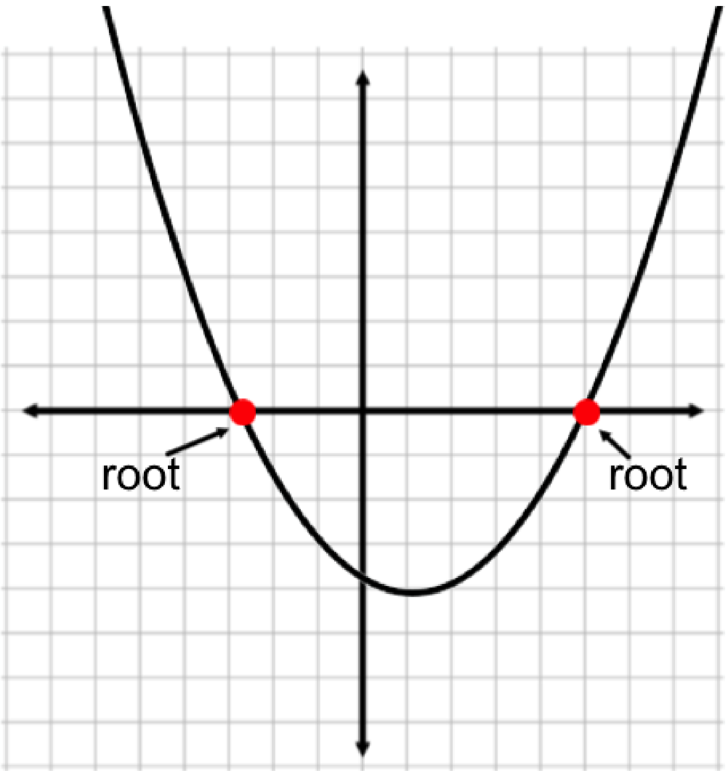
WHAT'S COVERED

In this lesson, you will learn how to solve a quadratic equation using the Zero Factor Property of Multiplication. Specifically, this lesson will cover:

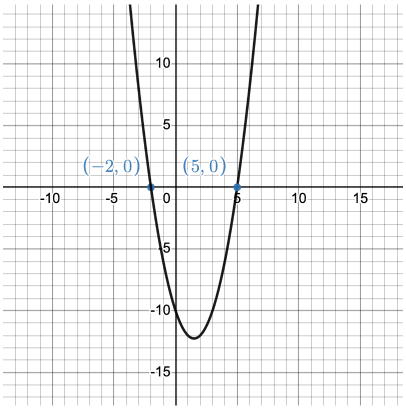
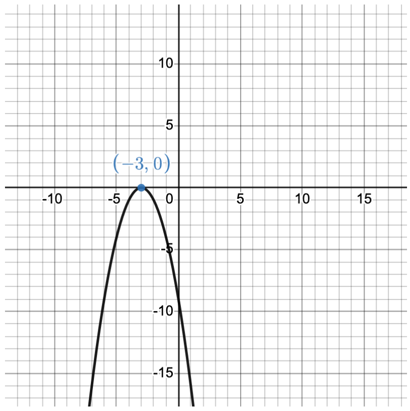
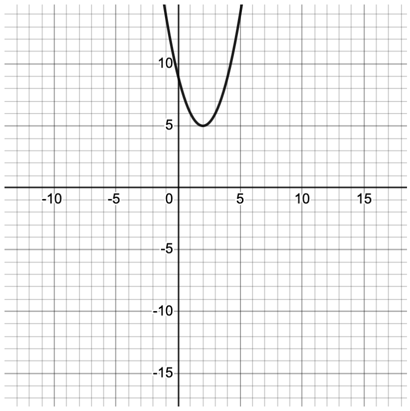
1. Solutions to Quadratic Equations

There are several ways we can talk about solutions to quadratic equations. Solutions are also called roots or zeros because they represent x-values to the equation that makes y equal to zero. Below is a graphical representation of solutions to a quadratic equation:



HINT

Quadratic equations will have either one, two, or zero real solutions. We can relate this to the graphical interpretation of solutions. A parabola will intersect the x-axis no more than two times. If a parabola never intersects the x-axis, it has no real solutions.

Two Solutions	One Solution	No Solutions
		
Solutions: (-2, 0) and (5, 0)	Solutions: (-3, 0)	No Real Solutions

2. Zero Factor Property of Multiplication

If any factor of an algebraic expression equals zero, the entire expression has a value of zero. This is because any quantity multiplied by zero is zero. This fact comes in handy when solving quadratic equations written in standard form. We can simply set each factor equal to zero, and solve simpler equations to find solutions to the quadratic.

EXAMPLE Find the solutions for the quadratic equation $x^2 - 4x - 21 = 0$.

$x^2 - 4x - 21 = 0$	Factor the left side of the equation by finding two numbers that multiply to -21 and add to -4
$(x+3)(x-7) = 0$	Set each factor equal to zero
$x+3=0,\ x-7=0$	Solve for x in each factor
$x = -3,\ x = 7$	Our solutions

By setting each factor equal to zero and solving for x, we have found two solutions to the quadratic. When $x = -3,$ the quadratic will equal zero:

$$(x+3)(x-7)=0$$
$$(-3+3)(-3-7)=0$$
$$(0)(-10)=0$$
$$0=0$$

Similarly, when $x = 7,$ the quadratic will equal zero:

$$(x+3)(x-7)=0$$
$$(7+3)(7-7)=0$$
$$(10)(0)=0$$
$$0=0$$

BIG IDEA

Use the Zero Factor Property of Multiplication to solve quadratic equations by setting each factor equal to zero. Solving for x when each factor is set equal to zero will reveal solutions to the quadratic.

3. Solving a Quadratic with No Constant Term

When a quadratic is set equal to zero, but has no constant term, making use of the Zero Factor Property is still helpful, but there is some work that needs to be done first. Without the constant term, we know that all of the terms in the quadratic share a factor of x. This means we can factor it out, and then apply the Zero Factor Property.

EXAMPLE Find the solutions for the quadratic equation $2x^2-6x=0$.

$2x^2-6x=0$	Factor out x
$x(2x-6)=0$	Set each factor equal to 0
$x=0,\ 2x-6=0$	Solve for x in each factor
$x=0,\ 2x=6$	Further simplify the second factor
$x=0,\ x=3$	Our solutions

Again, we can check our solutions by plugging these values back into the quadratic equation.

When $x=0$:

$$2x^2-6x=0$$
$$2(0)^2-6(0)=0$$
$$2(0)-0=0$$
$$0-0=0$$
$$0=0$$

When $x=3$:

$$2x^2-6x=0$$
$$2(3)^2-6(3)=0$$
$$2(9)-18=0$$
$$18-18=0$$
$$0=0$$

4. Solving a Quadratic with No x-Term

If a quadratic equation is missing an x-term, using the Zero Factor Property is not as helpful. Instead, we can solve using standard algebraic techniques for solving any equation. The important thing to remember here is that when we apply the square root to a quantity squared, we must include both positive and negative values of the root because a negative value squared is also positive.

→ EXAMPLE Find solutions to the quadratic equation $x^2 - 7 = 0$.

$x^2 - 7 = 0$	Move the constant to the right side by adding 7 to both sides
$x^2 = 7$	Apply square root to both sides
$\sqrt{x^2} = \sqrt{7}$	Evaluate
$x = \pm\sqrt{7}$	Don't forget to include both positive and negative solutions
$x = -\sqrt{7},\ x = \sqrt{7}$	Our solutions

HINT

Be sure to include plus or minus when taking the square root in equations. In our example above, both $-\sqrt{7}$ and $\sqrt{7}$ evaluate to 7 when squared.

SUMMARY

To find the **solutions to quadratic equations** algebraically, let y equal 0 and solve the equation for x. The solutions to a quadratic equation are also called the roots or zeros. On a graph, the solutions are the x-intercepts of the parabola. The **zero factor property of multiplication** says that if any factor is equal to zero, then the entire expression is equal to zero. This property is used when solving a quadratic equation by factoring and can be used when **solving a quadratic with no constant term** and **solving a quadratic with no x-term**.