Solving Single Step Equations

by Sophia

WHAT'S COVERED

This tutorial covers solving single step equations, through the definition and discussion of:

1. What Is an Equation?

An **equation** is a mathematical statement that two expressions, or quantities, have the same value.

EXAMPLE Below is an equation wherein both sides have a value of 15. The equal sign is used to state that the two quantities are equal, or have the same value. The left side is equal to 15 and the right side is also equal to 15.

$$7+8=10+5$$
$$15=15$$

Equations are widely used throughout mathematics, statistics, business, and sciences such as physics. You are also solving equations in daily life, such as when you make purchases and determine income. Being able to solve simple and complex equations is a fundamental mathematical skill.

TERM TO KNOW

Equation
A mathematical statement that two expressions or quantities have the same value

2. Variables and Properties of Equality

A **variable**, represented with a letter in an equation, is an unknown value that you are trying to find.

There are several properties of equality that help to solve equations or determine the variable in an equation.

Property of Equality	FORMULA TO KNOW	Description
Addition Property of Equality	$\text{If } a=b\text{, then } a+c=b+c$	The addition property states that if a equals b, and c is any number, then a plus c is equal to b plus c. Adding c on both sides of the equation still provides a true statement.
Subtraction Property of Equality	$\text{If } a=b\text{, then } a-c=b-c$	The subtraction property states that if a equals b, and c is any number, then a minus c is equal to b minus c. Subtracting c from both sides of the equation still provides a true statement.
Multiplication Property of Equality	$\text{If } a=b\text{, then } a\times c=b\times c$	The multiplication property states that if a is equal to b, and c is any number, then a times c is equal to b times c. Multiplying by c on both sides of the equation still provides a true statement.
Division Property of Equality	$\text{If } a=b\text{, then } a\div c=b\div c$ $(c \text{ must be a non}-\text{zero number})$	The division property states that if a is equal to b, and c is any *non-zero number*—you can't divide a number by 0—then a divided by c is equal to b divided by c. Dividing by c on both sides of the equation still provides a true statement.

BIG IDEA

In general, these properties state that whatever is done on one side of the equal sign must be done on the other side to maintain an equation, or a true statement.

TERM TO KNOW

Variable
An unknown value that we are trying to find

3. Inverse Operations

You can also use inverse operations to solve equations. Inverse operations are pairs of operations that undo each other, or cancel each other out. Addition and subtraction are a pair of inverse operations, because they undo each other, or cancel each other out.

EXAMPLE If you start with 3 and add 5, this equals 8. However, if you start with 8 and subtract 5, you are back to your original value of 3. Therefore, adding 5 and subtracting 5 undo each other. Similarly, you can see this is true if you look at 12 minus 8, which equals 4. However, if you start with 4 and add 8, you are back to your original value of 12.

$3+5=8 \quad 8-5=3$
$12-8=4 \quad 4+8=12$

Multiplication and division are also inverse operations because they cancel each other out.

EXAMPLE We know that 2 multiplied by 7 equals 14, but 14 divided by 7 will bring you back to your original value of 2. Therefore, multiplying by 7 and dividing by 7 undo each other. Similarly, 18 divided by 6 equals 3, but if you start with 3 and multiply by 6, you are back to your original value of 18.

$2\times 7=14 \quad 14\div 7=2$
$18\div 6=3 \quad 3\times 6=18$

Lastly, squaring and taking the square root are also inverse operations, because they cancel each other out.

EXAMPLE If you start with 7 and square it, this equals 49, and if you take the square root of 49, you are back to your original value of 7. Similarly, If you take the square root of 36, this equals 6, and if you start with 6 and square it, you are back to your original value of 36.

$7^2=49 \quad \sqrt{49}=7$
$\sqrt{36}=6 \quad 6^2=36$

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4. Isolating the Variable

Finding the solution or solving most equations involves isolating a variable. To do this, you want to rearrange your equation so that the variable, or the unknown quantity, is by itself on one side of the equation, and everything else is on the other side. To rearrange your equation, you can use the operations that are the inverse of the operations appearing in the equation.

EXAMPLE Suppose you want to solve the following equation: $x+10=25$.

To do this, you'll need to isolate the variable x on the left side of the equation. Because you are adding 10 to the variable x, you need to use the inverse operation to addition, which is subtraction, to cancel out the plus 10. Therefore, you're going to subtract 10 on the left side of the equation, which means you also need to subtract 10 on the right side of the equation. Remember, whatever is done on one side of the equal sign must be done on the other side in order to maintain an equation, or a true statement.

$x+10-10=25-10$

Then you can simplify each side.

$x=15$

Now you have the variable x on the left and 15 on the right, so your solution is x equals 15.

TRY IT

Consider the equation $\frac{1}{3}x = 4$.

Solve for x.

+

$\frac{1}{3}x = 4$	Our Equation
$\frac{3}{1} \times \frac{1}{3}x = 4 \times \frac{3}{1}$	Because the x is multiplied by 1/3, we need to use the inverse operation of division to cancel it out. This may look complicated, so another way to cancel it out is to multiply it by the reciprocal, 3/1.
$\frac{3}{3}x = \frac{12}{1}$	Multiplying by 3/1 on the left side of the equation equals 3/3 times x. Multiplying by 3/1 on the right side equals 12/1.
$1x = 12$	Simplify both sides of the equation.
$\frac{1x}{1} = \frac{12}{1}$	Divide by one on both sides of the equation to get the final answer.
$x = 12$	Our Solution

IN CONTEXT

Suppose Jaime scored 18 goals during 9 games of soccer. He wants to know how many goals he scored, on average, per game. You can write an equation to represent the situation.

You can multiply the 9 games of soccer by x, your variable, which represents how many goals he scored per game. Lastly, you know that this will equal the total number of goals he scored, 18.

$9x = 18$

Because the x is multiplied by 9, you need to divide by 9 on both sides of the equation to isolate the x. This simplifies to be x on the left side of the equation, and 2 on the right side of the equation. Therefore, your solution is x equals 2, which means that Jaime scored, on average, 2 goals per game.

$$\frac{9x}{9} = \frac{18}{9}$$

$$x = 2$$

SUMMARY

Today you learned that an **equation** is a mathematical statement that two expressions, or quantities, have the same value. A **variable**, which is represented with a letter in an equation, is an unknown value that you are trying to find or solve for. You also learned about several **properties of equality**, which can be used to help solve for a variable in an equation. Lastly, you learned that to solve for a variable, you want to **isolate the variable** on one side of the equation and move everything else to the other side, and you can do this using **inverse operations**.

Source: This work is adapted from Sophia author Colleen Atakpu.

TERMS TO KNOW

Equation
A mathematical statement that two expressions or quantities have the same value.

Variable
An unknown value that we are trying to find.

FORMULAS TO KNOW

Addition Property of Equality
$If\ a = b,\ then\ a + c = b + c$

Division Property of Equality
$If\ a = b,\ then\ a \div c = b \div c$

Multiplication Property of Equality
$If\ a = b,\ then\ a \times c = b \times c$

Subtraction Property of Equality
$If\ a = b,\ then\ a - c = b - c$