Solving Single-Step Equations

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to identify the operation needed to solve a single-step equation. Specifically, this lesson will cover:

1. Equations

An **equation** is a mathematical statement that expressions or quantities have the same value.

EXAMPLE The equation $x + 13 = 20$ is saying that the quantity *x* plus 13 has the same value as 20.

In the example above, we have a variable *x*. A variable is a value that can change. But when it is in an equation, there is then a certain value that makes the statement true. Sometimes, *x* could have several values that make it true. However, it is not always the case that *x* can be whatever it wants to be. It has to fit that particular equation.

Equations can also be written in the other direction and is known as the rule of symmetry.

EXAMPLE In the previous example, we could say $20 = x + 13$.

TERM TO KNOW

Equation

A mathematical statement that two quantities or expressions are equal in value.

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2. Solving Single-Step Equations

Solving linear equations is an important and fundamental skill in algebra. In algebra, we are often presented with a problem where the answer is known, but part of the problem is missing. The missing part of the problem is what we seek to find.

$$4x + 16 = -4$$

Notice the above problem has a missing part, or unknown, that is marked by *x*. If we are given that the solution to this equation is -5, it could be plugged into the equation, replacing the *x* with -5. This is shown below:

↱ EXAMPLE

$4x+16=-4$ Replace x with -5

$4(-5)+16=-4$ Multiply $4(-5)$

$-20+16=-4$ Add $-20+16$

$-4=-4$ True!

Now the equation comes out to a true statement! Notice also that if another number, for instance, 3, was plugged in, we would not get a true statement.

↱ EXAMPLE

$4x+16=-4$ Replace x with 3

$4(3)+16=-4$ Multiply $4(3)$

$12+16=-4$ Add $12+16$

$28\neq-4$ False!

Due to the fact that this is not a true statement, this demonstrates that 3 is not the solution. However, depending on the complexity of the problem, this “guess and check” method is not very efficient. Thus, we take a more algebraic approach to solving equations. Here we will focus on what are called “one-step equations” or equations that only require one step to solve. While these equations often seem very fundamental, it is important to master the pattern for solving these problems so we can solve more complex problems.

2a. Addition Problems

To solve equations, the general rule is to do the opposite. For addition problems, this means that we will do the opposite operation, which is subtraction.

↱ EXAMPLE

$x+7=-5$ The 7 is added to the x, so subtract 7 from both sides

$-7\quad -7$ The 7's on the left side cancel, leaving x. Subtract 7 from -5

$x=-12$ Our Solution

The same process is used in each of the following examples.

↱ EXAMPLE

$$\begin{array}{r} 4+x=8 \\ \underline{-4\quad\ -4} \\ x=4 \end{array} \qquad \begin{array}{r} 7=x+9 \\ \underline{-9\quad\ -9} \\ -2=x \end{array} \qquad \begin{array}{r} 5=8+z \\ \underline{-8\quad -8} \\ -3=\quad z \end{array}$$

HINT

Notice that when we subtracted 7 from the left side to cancel this value, we also had to subtract 7 from the right side. This is referred to as the **rule of equality**. This is true for any operation; if you perform an operation on one side of the equation, you must do the same operation on the other side.

TERM TO KNOW

Rule of Equality

Any operation performed on one side of the equation must be performed on the other side, in order to keep quantities equal in value.

2b. Subtraction Problems

In a subtraction problem, we get rid of negative numbers by adding them to both sides of the equation.

↳ EXAMPLE

$x-5=4$ The 5 is subtracted from *x*, so add 5 to both sides

$\underline{\;+5\;\;+5\;}$ The 5's on the left side cancel, leaving *x*. Add 4 and 5 together

$x=9$ Our Solution

The same process is used in each of the following examples. Notice that each time we are getting rid of a negative number by adding.

↳ EXAMPLE

$$\begin{array}{rcl} -6+x & = & -2 \\ \underline{+6\;\;\;\;\;} & & \underline{+6} \\ x & = & 4 \end{array} \qquad \begin{array}{rcl} -10 & = & x-7 \\ \underline{+7} & & \underline{\;\;+7} \\ -3 & = & x \end{array} \qquad \begin{array}{rcl} 5 & = & -8+x \\ \underline{+8} & & \underline{+8\;\;\;\;\;} \\ 13 & = & x \end{array}$$

2c. Multiplication Problems

With a multiplication problem, we get rid of the number by dividing both sides. Note that we are showing division with a fraction bar, for instance, $\frac{24}{6}=4$. Using a fraction bar is the same as division. $\frac{15}{3}=5$ is the same as $15 \div 3 = 5$.

↳ EXAMPLE

$4x=20$ Variable is multiplied by 4, so divide both sides by 4

$\overline{4}\;\;\overline{4}$ The 4's on the left side cancel, leaving *x*. Divide 20 by 4

$x=5$ Our Solution

With multiplication problems, it is very important that care is taken with signs. If *x* is multiplied by a negative then we will divide by a negative.

↳ EXAMPLE

$-5x=30$ Variable is multiplied by -5, so divide both sides by -5

$\overline{-5}\;\;\overline{-5}$ The -5's on the left side cancel, leaving *x*. Divide 30 by -5

$x=-6$ Our Solution

The same process is used in each of the following examples. Notice how negative and positive numbers are handled as each problem is solved.

↳ EXAMPLE

$$\frac{8x}{8} = \frac{-24}{8} \qquad \frac{-4x}{-4} = \frac{-20}{-4} \qquad \frac{42}{7} = \frac{7x}{7}$$

$$x=-3 \qquad\qquad x=5 \qquad\qquad 6=x$$

2d. Division Problems

In division problems, we get rid of the denominator by multiplying on both sides.

EXAMPLE

$$\frac{x}{5} = -3$$ Variable is divided by 5, so multiply both sides by 5

$$(5)\frac{x}{5} = -3(5)$$ The 5's on the left side cancel, leaving *x*. Multiply -3 and 5

$$x = -15$$ Our Solution

The same process is used in each of the following examples.

EXAMPLE

$$\frac{x}{-7} = -2$$
$$(-7)\frac{x}{-7} = -2(-7)$$
$$x = 14$$

$$x \div 8 = 5$$
$$(x \div 8) \cdot 8 = 5 \cdot 8$$
$$x = 40$$

$$\frac{x}{-4} = 9$$
$$(-4)\frac{x}{-4} = 9(-4)$$
$$x = -36$$

The process described above is fundamental to solving equations. Once this process is mastered, the problems we will see have several more steps. These problems may seem more complex, but the process and patterns used will remain the same.

BIG IDEA

To solve single-step equations, first identify the operation being applied to the variable. To isolate the variable, apply the inverse operation. Addition and subtraction are inverses of each other; multiplication and division are inverses of each other.

SUMMARY

Solving single-step equations involves isolating the variable that you're trying to solve for by using an inverse operation. Any operation that you do on one side of the equation needs to be done on the other side. You can always check your solution by substituting it in for the variable in your original equation and seeing if the statement holds true. This is good practice to get into when you're doing simple equations because when you do things more complicated, it's more likely that you're going to be making a mistake.

TERMS TO KNOW

Equation
A mathematical statement that two quantities or expressions are equal in value.

Rule of Equality
Any operation performed on one side of the equation must be performed on the other side, in order to keep quantities equal in value.