Special Products of Binomials

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WHAT'S COVERED

In this lesson, you will learn how to expand a special product binomial. Specifically, this lesson will cover:

1. FOIL Review

Our special products of binomials come through by noticing how expressions are simplified in binomial multiplication. Binomial multiplication is often modeled through a process known as FOIL, which is an acronym that stands for First, Outside, Inside, Last. It describes an order to multiply terms that make up the two factors in binomial multiplication.

EXAMPLE Multiply $(x+2)(x-3)$.

$(x+2)(x-3)$	Multiply first terms: $x \cdot x = x^2$
x^2	Multiply outside terms: $x \cdot -3 = -3x$
x^2-3x	Multiply inside terms: $2 \cdot x = 2x$
$x^2-3x+2x$	Multiply last terms: $2 \cdot -3 = -6$
$x^2-3x+2x-6$	Combine like terms
x^2-x-6	Our solution

2. Square of a Binomial Sum

Take a look at what happens when we expand a binomial sum that is being squared:

EXAMPLE Multiply $(x+3)^2$.

$(x+3)^2$	Rewrite as multiplication of two factors, $(x+3)$
$(x+3)(x+3)$	FOIL
$x^2+3x+3x+9$	Combine like terms
x^2+6x+9	Our solution

Note the relationship between the constant in our binomial sum (3), the coefficient of the x-term (6), and the constant term in the expanded trinomial (9). See if the relationship you spotted holds true in another example:

EXAMPLE Multiply $(x+5)^2$.

$(x+5)^2$	Rewrite as multiplication of two factors, $(x+5)$
$(x+5)(x+5)$	FOIL
$x^2+5x+5x+25$	Combine like terms
$x^2+10x+25$	Our solution

What is the relationship between 5, 10, and 25? If we take the constant from the binomial sum, 5, and double this number, we get the coefficient of x-term, 10. And if we square 5, we get 25, which is the constant term in the expanded trinomial. This holds true for all squares of binomial sums.

FORMULA TO KNOW

Square of a Binomial Sum

$$(x+a)^2 = x^2+2ax+a^2$$

Test it out with a few more examples:

Original Expression	Coefficient of x-term	Constant	Expanded Expression
$(x+2)^2$	$2\cdot 2=4$	$2^2=4$	x^2+4x+4
$(x+7)^2$	$2\cdot 7=14$	$7^2=49$	$x^2+14x+49$
$(x+12)^2$	$2\cdot 12=24$	$12^2=144$	$x^2+24x+144$

3. Square of a Binomial Difference

Next, let's take a look at similar expressions, except these examples involve subtraction instead of addition:

EXAMPLE Multiply $(x-3)^2$.

$(x-3)^2$	Rewrite as multiplication of two factors, $(x-3)$
$(x-3)(x-3)$	FOIL
$x^2-3x-3x+9$	Combine like terms
x^2-6x+9	Our solution

Note that the only distinction here is that the x-term is a negative 6*x*, because we subtracted 3 from *x* in the binomial. We still have +9 because -3 times -3 is a positive number. Let's take a look at another example:

EXAMPLE Multiply $(x-5)^2$.

$(x-5)^2$	Rewrite as multiplication of two factors, $(x-5)$
$(x-5)(x-5)$	FOIL
$x^2-5x-5x+25$	Combine like terms
$x^2-10x+25$	Our solution

We can generalize this as the square of a binomial difference:

FORMULA TO KNOW

Square of a Binomial Difference

$(x-a)^2=x^2-2ax+a^2$

Test it out with a few more examples:

Original Expression	Coefficient of x-term	Constant	Expanded Expression
$(x-4)^2$	$2\cdot -4 = -8$	$(-4)^2=16$	$x^2-8x+16$
$(x-8)^2$	$2\cdot -8 = -16$	$(-8)^2=64$	$x^2-16x+64$
$(x-10)^2$	$2\cdot -10 = -20$	$(-10)^2=100$	$x^2-20x+100$

BIG IDEA

The expanded form for the square of a binomial sum and the square of a binomial difference are also referred to as **perfect square trinomials** because they consist of three terms, which can be simplified into an expression squared.

TERM TO KNOW

Perfect Square Trinomial

A polynomial with three terms, we can be simplified as a binomial squared, $(x+a)^2$ or $(x-a)^2$.

4. Difference of Squares

So far we have discussed special products involving a binomial squared, where a is either positive or negative. We can have a binomial sum $(x+a)^2$ or a binomial difference $(x-a)^2$. Now, we are going to consider the case where we have a binomial sum multiplied by a binomial difference.

EXAMPLE Multiply $(x-3)(x+3)$.

$(x-3)(x+3)$	FOIL
$x^2+3x-3x-9$	Combine like terms
x^2-9	Our solution

There are a couple of things to note in this example:

- The x-terms canceled, because the coefficients were opposites of each other.
- The constant term here is negative, because the product of a positive and negative number is always negative.
- The constant term is also a perfect square.

We can generalize this as a **difference of squares**.

FORMULA TO KNOW

Difference of Squares

$(x+a)(x-a) = x^2-a^2$

Test it out with a few more examples:

Original Expression	Coefficient of x-term	Constant	Expanded Expression
$(x-2)(x+2)$	$-2+2=0$	$-2\cdot2=-4$	x^2-4
$(x-7)(x+7)$	$-7+7=0$	$-7\cdot7=-49$	x^2-49
$(x-13)(x+13)$	$-13+13=0$	$-13\cdot13=-169$	x^2-169

TERM TO KNOW

Difference of Squares

Two squared terms separated by subtraction, $x^2 - a^2$, which can be expressed as $(x+a)(x-a)$.

SUMMARY

A **foil review** reminds us that FOIL is the acronym used to remember the way to multiply two binomials. It stands for First, Outside, Inside, Last. Recognizing special products of binomials can help make factoring and FOILing easier. The three special products are **square of a binomial sum**: $(x+a)^2$, **square of a binomial difference**: $(x-a)^2$, and the **difference of squares**: $(x+a)(x-a)$. A perfect square trinomial is a polynomial with three terms that can be simplified as a binomial squared.

TERMS TO KNOW

Difference of Squares

Two squared terms separated by subtraction, $x^2 - a^2$, which can be expressed as $(x+a)(x-a)$.

Perfect Square Trinomial

A polynomial with three terms, which can be simplified as a binomial squared, $(x+a)^2$ or $(x-a)^2$.

FORMULAS TO KNOW

Difference of Squares

$(x+a)(x-a) = x^2 - a^2$

Square of a Binomial Difference

$(x-a)^2 = x^2 - 2ax + a^2$

Square of a Binomial Sum

$(x+a)^2 = x^2 + 2ax + a^2$

