

# Standard Normal Distribution

by Sophia



## WHAT'S COVERED

This tutorial will cover the topic of standard normal distribution. Our discussion breaks down as follows:

### 1. Standard Normal Distribution

### 2. Standard Normal Tables

#### 2a. Negative Z-Scores

#### 2b. Positive Z-Scores

### 3. Different Cases

## 1. Standard Normal Distribution

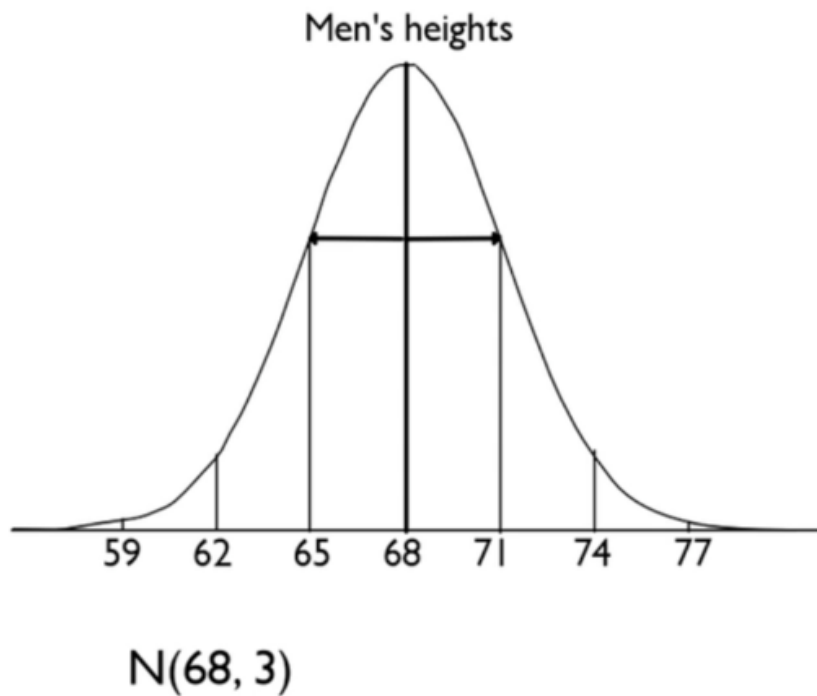
The **standard normal distribution** is a specific kind of normal distribution that uses standard scores. Standard scores are also known as z-scores.

In the standard normal distribution, the mean will be zero, and the standard deviation is one.

⇒ **EXAMPLE** Here is an example to show how to use standard normal distribution. Men's heights are normally distributed with a mean of 68 inches, which is five feet eight inches, and a standard deviation of 3 inches.

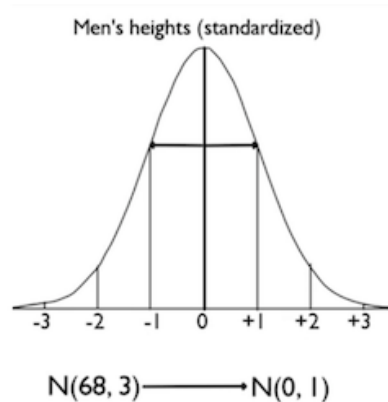
What percent of men are over six feet (or 72 inches) tall?

First, recall the 68-95-99.7 Rule, which says that 68% of the data points fall within one standard deviation of the mean. That means that 68% of men's heights will fall within three inches of 68. 95% will fall within two standard deviations, and 99.7% percent will fall within three standard deviations.



Where is 72 inches? It falls between the first and second standard deviation above the mean, and the goal was to find the number of men above that height. So, because the answer is not on an integer standard deviation, it becomes an issue. How do you find the answer?

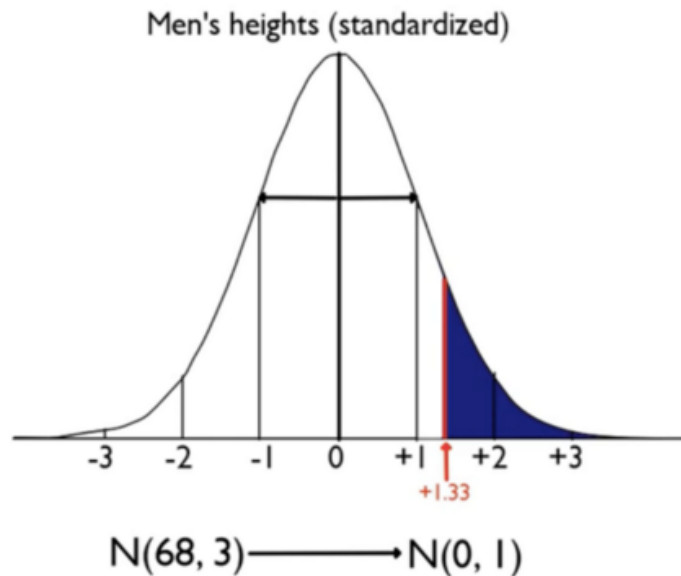
To solve this challenge, take these heights and standardize them by turning these values into z-scores. Z-scores are how many standard deviations away from the mean an observation is. In the above distribution, 71 is one standard deviation above the mean, so in our new graph, it will be marked as +1. Next, 74 is two standard deviations above the mean, so it will be marked as +2. The rest of the numbers will follow, resulting in a graph that looks like this:



In this graph, the 68 has been marked as 0, and all the other numbers marked as their respective integer number of standard deviations away from the mean. But remember, 72 wasn't an integer number of standard deviations. It was somewhere between one and two standard deviations away. So we can calculate the exact z-score for that cutoff point of 72 inches by using the z-score formula.

$$z = \frac{72 - 68}{3} = \frac{4}{3} = 1.33$$

It's at 1.33 standard deviations above the mean, and our goal will be to find the percent of values that are above this score.



#### TERM TO KNOW

##### Standard Normal Distribution

A normal distribution of z-scores. The mean is zero, and the standard deviation is one.

## 2. Standard Normal Tables

To solve the problem we were addressing above, you need a **standard normal table**, or **z-table**. A standard normal table is a table of probabilities that lie below particular z-scores; it's a table that calculates the percent of values below a particular z-score. With these tables, you can find the percent of values that fall at or below a particular z-score.



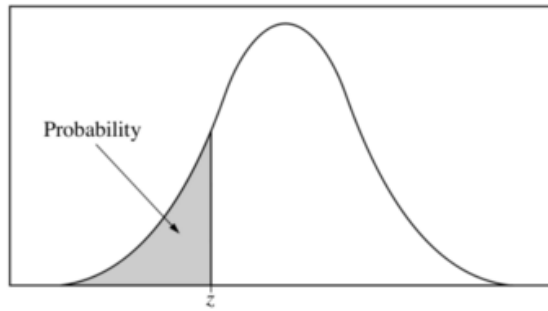
#### TERM TO KNOW

##### Standard Normal Table/Z-Table

A table that calculates the percent of values below a particular z-score.

### 2a. Negative Z-Scores

Negative z-scores correspond to values that are below the mean. To find the probability of a negative z-score, we need to use the z-table that shows negative values. In the table below, the left column shows negative z-scores that fall to the *left* of the mean. Also, notice in the graph that the values are always below the mean.

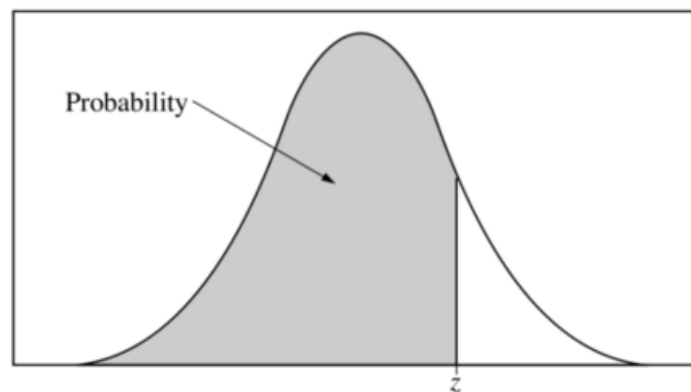


$z$	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379

-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3829
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

## 2b. Positive Z-Scores

Positive z-scores correspond to values that are above the mean. To find the probability of a positive z-score, we need to use the z-table that shows positive values. In the table below, the left column shows positive z-scores that fall to the *right* of the mean. Also, notice in the graph that the values are always above the mean.



z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852

0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997

You're only going to need to use one of these tables at a time. Because your z-score was positive 1.33, you're going to use the positive z-score table.

- The column on the far left represents the tenths place of your z-score. Your z-score was 1.33, so you're going to find 1.3 as your tenths.

- The row across the top represents the hundredths place of your z-score. Your z-score was 1.33, so you're going to find 0.03 as your hundredths.

Then look in the table for the value that corresponds to the tenths place of 1.3 and the hundredths place of 0.03:

		Hundredths					
Tenths	z	+0.00	+0.01	+0.02	0.03	0.04	+0.05
	0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199
	0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
	0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
	0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368
	0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736
	0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088
	0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422
	0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734
	0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023
	0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289
	1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531
	1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749
	1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944
	1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115
	1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265
	1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394
	1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505
	1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599

However, this is going to give you the amount of area *below*, meaning the percent of men shorter than 72 inches tall. A probability of 0.9082 tells us that 90.82% of men are *shorter* than six feet tall.

Since the original question was what percent of men are *taller* than six feet, subtract 90.82% from 100%. You should find that 9.18% of men have heights of over 72 inches.

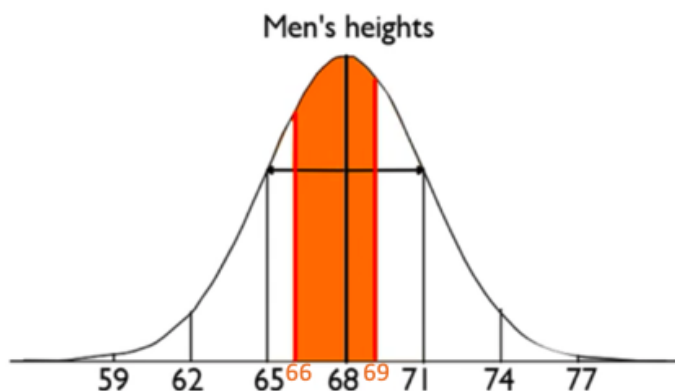
### 3. Different Cases

We can use the z-table to find the probability for different scenarios:

- Percent below a particular value: This is the easiest to find because the table gives you the percentage below a value.
- Percent above a particular value: This can be found by taking the table value and subtracting from 100%.
- Percent between two values: This will require a few extra steps.

⇒ **EXAMPLE** Men's heights are normally distributed with a mean of 68 inches, which is five feet eight inches, and a standard deviation of 3 inches.

What percent of men are between 5'6" and 5'9", or between 66 and 69 inches tall?



The orange area in the above graph is what you're looking for. First, you need to calculate the z-scores of each value.

*Height of 66 inches*

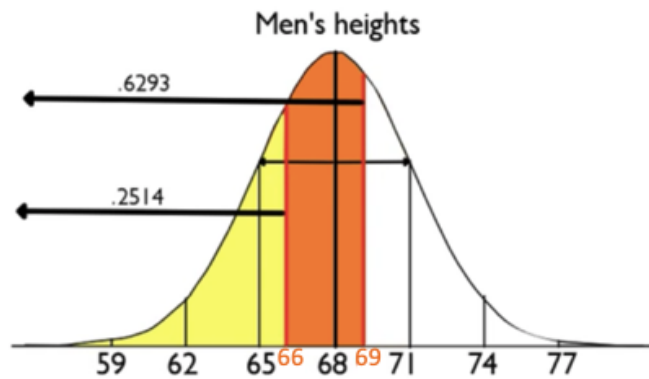
$$z = \frac{66 - 68}{3} = \frac{-2}{3} = -0.67$$

*Height of 69 inches*

$$z = \frac{69 - 68}{3} = \frac{1}{3} = 0.33$$

The z-scores are negative 0.67 and positive 0.33. Next, find the table value that corresponds to positive 0.33, which is 0.6293 from the table. Then, find the table value for negative 0.67, which is 0.2514.





$$\text{Orange Area} = P(z < 0.33) - P(z < -0.67)$$

$$\text{Orange Area} = 0.6293 - 0.2514$$

$$\text{Orange Area} = 0.3779$$

To find just the orange area, we need to subtract the full area that corresponds to a z-score of 0.33, which was about 63%, and the area that corresponds to a z-score of -0.67, which was approximately 25%.

When you subtract them, you end up with about 0.6293 minus 0.2514, or 0.3779. This tells us that the orange area that contains 38% represents all men who fall in between those two heights.



## SUMMARY

It's possible to find the percent of values above or below a particular value using something other than the 68-95-99.7 Rule using the z-scores on the standard normal distribution. The standard normal table helps you find the percent of values below a particular z-score in order to calculate a percentage of values above or below a particular value, or between two values.

Good luck!

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## TERMS TO KNOW

### Standard Normal Distribution

A normal distribution of z-scores. The mean is zero, and the standard deviation is one.

### Standard Normal Table/z-table

A table that calculates the percent of values below a particular z-score.