Sophia

Sum of a Finite Geometric Sequence

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to find the sum of a finite geometric sequence. Specifically, this lesson will cover:

1. Formula for the Sum of a Finite Geometric Sequence

Consider the following geometric sequence: $\{24, 12, 6, 3\}$. To find the sum of this finite sequence, it is simple enough to concretely add together all of the terms:

$$24+12+6+3=45$$

However, there is a useful formula we can use in cases where this may be difficult to do, for example, if there were 100 terms to add.

FORMULA TO KNOW

Sum of a Finite Geometric Sequence

$$S_n = a_1 \cdot \left(\frac{1-r^n}{1-r}\right)$$

In this formula:

- S_n is the sum of n terms
- a_1 is the value of the first term
- r is the common ratio

- n is the number of terms

EXAMPLE Using the geometric sequence above $\{24, 12, 6, 3\}$, confirm that the sum of the four terms above is 45 by using the formula as well.

Let's identify a_1, r, and n:

- a_1: The first term is 24.
- r: The common ratio can be found by dividing any two consecutive numbers: $12 \div 24 = 0.5$.
- n: There are 4 terms, so n is 4.

Plug these values into the Sum of a Finite Geometric Sequnce formula:

$S_n = a_1 \cdot \left(\frac{1-r^n}{1-r}\right)$	Plug in $n = 4,\ a_1 = 24,\ r = 0.5$
$S_4 = 24 \cdot \left(\frac{1-0.5^4}{1-0.5}\right)$	Evaluate the exponent
$S_4 = 24 \cdot \left(\frac{1-0.0625}{1-0.5}\right)$	Simplify the numerator and denominator
$S_4 = 24 \cdot \left(\frac{0.9375}{0.5}\right)$	Evaluate the fraction
$S_4 = 24 \cdot (1.875)$	Multiply
$S_4 = 45$	Our solution

2. Geometric Sums when *r* is Greater than 1

Our formula works for any value of r, although when we are working through the calculations, it may seem as though something must be off. Don't worry, as long as you follow the steps properly, you should arrive at the correct sum.

EXAMPLE Find the sum of the geometric sequence $\{32, 48, 72, 108, 162\}$.

Again, let's identify a_1, r, and n:

- a_1: The value of the first term is 32.
- r: The common ratio is 1.5, which is found by dividing any two consecutive terms, such as $108 \div 72 = 1.5$.
- n: Since we have 5 terms in the sequence, $n = 5$.

Applying the formula, we can find the sum of these 5 terms:

$$S_n = a_1 \cdot \left(\frac{1-r^n}{1-r}\right)$$ Plug in $n = 5,\ a_1 = 32,\ r = 1.5$

$$S_5 = 32 \cdot \left(\frac{1-1.5^5}{1-1.5}\right)$$ Evaluate the exponent

$$S_5 = 32 \cdot \left(\frac{1-7.59375}{1-1.5}\right)$$ Simplify the numerator and denominator

$$S_5 = 32 \cdot \left(\frac{-6.59375}{-0.5}\right)$$ Evaluate the fraction

$$S_5 = 32 \cdot (13.1875)$$ Multiply

$$S_5 = 422$$ Our solution

Checking concretely: $32 + 48 + 72 + 108 + 162 = 422$

3. Geometric Sums when *r* is Less than 0

The formula even works if the common ratio is a negative number.

EXAMPLE Find the sum of the geometric sequence $\{6,\ -18,\ 54,\ -162,\ 486\}$.

First, identify a_1, r, and n:

- a_1: The first term is 6
- r: Here, the common ratio is -3, because we multiply 6 by -3 to get -18, -18 by -3 to get 54, and so on.
- n: There are 5 terms so $n = 5$

To find the sum of these 5 terms using the formula, we take the following steps:

$$S_n = a_1 \cdot \left(\frac{1-r^n}{1-r}\right)$$ Plug in $n = 5,\ a_1 = 6,\ r = -3$

$$S_5 = 6 \cdot \left(\frac{1-(-3)^5}{1-(-3)}\right)$$ Evaluate the exponent

$$S_5 = 6 \cdot \left(\frac{1-(-243)}{1-(-3)}\right)$$ Simplify the numerator and denominator

$$S_5 = 6 \cdot \left(\frac{244}{4}\right)$$ Evaluate the fraction

$$S_5 = 6 \cdot (61)$$ Multiply

$$S_5 = 366$$ Our solution

Confirming concretely: $6 - 18 + 54 - 162 + 486 = 366$

4. Partial Sums

We can even use the formula to find a partial sum of a geometric sequence. A partial sum means that we add some of the terms in the sequence, but not all of them.

EXAMPLE Find the sum of the terms 3 through 8 in the geometric sequence with 12 terms:

$$\{13, 26, 52, 104, 208, 416, 832, 1664, 3328, 6656, 13312, 26624\}$$

If we want to find the sum of terms 3 through 8, we would make the following adjustments to our formula:

- a_1: We will consider a_1 to be 52. Although it is the 3rd term in the sequence, it is the 1st term we wish to include in the sum.
- r: The common ratio is 2, since $26 \div 13 = 2$
- n: We will use $n = 6$ because the sum of terms 3 through 8 consists of 6 terms altogether: 3rd term, 4th term, 5th term, 6th term, 7th term, and 8th term.

Everything else about the formula remains the same:

$S_n = a_1 \cdot \left(\frac{1 - r^n}{1 - r}\right)$	Plug in $n = 6,\ a_1 = 52,\ r = 2$
$S_6 = 52 \cdot \left(\frac{1 - 2^6}{1 - 2}\right)$	Evaluate the exponent
$S_6 = 52 \cdot \left(\frac{1 - 64}{1 - 2}\right)$	Simplify the numerator and denominator
$S_6 = 52 \cdot \left(\frac{-63}{-1}\right)$	Evaluate the fraction
$S_6 = 52 \cdot (63)$	Multiply
$S_6 = 3276$	Our solution

Adding concretely: $52 + 104 + 208 + 416 + 832 + 1664 = 3276$

SUMMARY

In the **formula for the sum of a finite geometric sequence**, a_1 is the first term in the sequence, r is the common ratio between consecutive terms, and n is the number of terms. The formula for finding the sum of a finite geometric sequence can be used when r is both positive and negative, or the **geometric**

sum when *r* is greater than 1 or *r* is less than 0. The formula can also be used to calculate a **partial sum** of a finite or infinite sequence.

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FORMULAS TO KNOW

Sum of a Finite Geometric Sequence

$$S_n = a_1 \cdot \left(\frac{1-r^n}{1-r}\right)$$

