Synthetic Division & Long Division of Polynomials

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WHAT'S COVERED

In this lesson, you will learn how to identify the setup for synthetic division. Specifically, this lesson will cover:

1. Polynomial Long Division

Polynomial long division sounds like a complicated process, and it can be without an understanding of the similarities between the long division from your elementary school days. The process, or algorithm - the steps we take to arrive at our solution - is the same, we are just dealing with more complicated terms.

STEP BY STEP

1. Use the leading term of the dividend and divisor.
2. Divide the leading term of the dividend by the leading term of the divisor.
3. Multiply this result by the divisor and place below the dividend.
4. Subtract like terms.
5. Bring down the next term.
6. Repeat steps of dividing, multiplying, subtracting, and bringing down the next term.

EXAMPLE Divide $2x^3+3x^2-5x+12$ by $x+3$.

As you read through the steps, think about how the process is similar to numeric long division

$$x+3\overline{)\;2x^3+3x^2-5x+12}$$

Divide $\frac{2x^3}{x}=2x^2$

$$\begin{array}{r} 2x^2 \\ x+3\overline{)\;2x^3+3x^2-5x+12} \end{array}$$

Multiply $2x^2(x+3)=2x^3+6x^2$

$$\begin{array}{rl} & 2x^2 \\ x+3\,\big) & \overline{2x^3+3x^2-5x+12} \\ & (2x^3+6x^2) \end{array}$$

Subtract $(2x^3+3x^2)-(2x^3+6x^2) = -3x^2$

$$\begin{array}{rl} & 2x^2 \\ x+3\,\big) & \overline{2x^3+3x^2-5x+12} \\ & \underline{-(2x^3+6x^2)} \\ & -3x^2 \end{array}$$

Bring down next term, -5*x*

$$\begin{array}{rl} & 2x^2 \\ x+3\,\big) & \overline{2x^3+3x^2-5x+12} \\ & \underline{-(2x^3+6x^2)} \\ & -3x^2-5x \end{array}$$

Repeat process

At this point, we repeat our steps of dividing, multiplying, subtracting, and bringing down the next term:

$$\begin{array}{rl} & 2x^2 \\ x+3\,\big) & \overline{2x^3+3x^2-5x+12} \\ & \underline{-(2x^3+6x^2)} \\ & -3x^2-5x \end{array}$$

Divide $\frac{-3x^2}{x} = -3x$

$$\begin{array}{rl} & 2x^2-3x \\ x+3\,\big) & \overline{2x^3+3x^2-5x+12} \\ & \underline{-(2x^3+6x^2)} \\ & -3x^2-5x \end{array}$$

Multiply $-3x(x+3) = -3x^2-9x$

$$\begin{array}{rl} & 2x^2-3x \\ x+3\,\big) & \overline{2x^3+3x^2-5x+12} \\ & \underline{-(2x^3+6x^2)} \\ & -3x^2-5x \\ & (-3x^2-9x) \end{array}$$

Subtract $(-3x^2-5x)-(-3x^2-9x) = 4x$

$$\begin{array}{rl} & 2x^2-3x \\ x+3\,\big) & \overline{2x^3+3x^2-5x+12} \\ & \underline{-(2x^3+6x^2)} \\ & -3x^2-5x \\ & \underline{-(-3x^2-9x)} \\ & 4x \end{array}$$

Bring down next term, +12

$$
\begin{array}{r}
2x^2-3x \\
x+3\overline{)\,2x^3+3x^2-5x+12} \\
\underline{-(2x^3+6x^2)} \\
-3x^2-5x \\
\underline{-(-3x^2-9x)} \\
4x+12
\end{array}
$$

Repeat process

We repeat the cycle of steps once again:

$$
\begin{array}{r}
2x^2-3x \\
x+3\overline{)\,2x^3+3x^2-5x+12} \\
\underline{-(2x^3+6x^2)} \\
-3x^2-5x \\
\underline{-(-3x^2-9x)} \\
4x+12
\end{array}
$$

Divide $\frac{4x}{x}=4$

$$
\begin{array}{r}
2x^2-3x+4 \\
x+3\overline{)\,2x^3+3x^2-5x+12} \\
\underline{-(2x^3+6x^2)} \\
-3x^2-5x \\
\underline{-(-3x^2-9x)} \\
4x+12
\end{array}
$$

Multiply $4(x+3)=4x+12$

$$
\begin{array}{r}
2x^2-3x+4 \\
x+3\overline{)\,2x^3+3x^2-5x+12} \\
\underline{-(2x^3+6x^2)} \\
-3x^2-5x \\
\underline{-(-3x^2-9x)} \\
4x+12 \\
(4x+12)
\end{array}
$$

Subtract $(4x+12)-(4x+12)$

$$
\begin{array}{r}
2x^2-3x+4 \\
x+3\overline{)\,2x^3+3x^2-5x+12} \\
\underline{-(2x^3+6x^2)} \\
-3x^2-5x \\
\underline{-(-3x^2-9x)} \\
4x+12 \\
\underline{-(4x+12)} \\
0\ \text{Remainder}
\end{array}
$$

Our solution with 0 Remainder

This tells us that the quotient is $2x^2-3x+4$ with zero remainder.

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2. Synthetic Division

Synthetic division is a process that is designed to be easier than polynomial long division. It works primarily with the coefficients to the term, and simplifies the steps in the algorithm to arrive at the same solution.

STEP BY STEP

1. Set up synthetic division using coefficients of the dividend polynomial and the constant term of the divisor.
2. Bring the first number in the box down below.
3. Multiply this number by the value to the left of the box.
4. Write the product below the next number in the box.
5. Add vertically, and write this below the box.
6. Repeat these steps until there are no more operations left to do.

We are going to do the same division example as above, but set the problem up using synthetic division:

EXAMPLE Divide $2x^3+3x^2-5x+12$ by $x+3$.

Step 1: First, use the coefficients of the dividend (what is being divided) and write them inside of a box, as shown below:

$$\begin{array}{|rrrr} 2 & 3 & -5 & 12 \\ \hline \end{array}$$

Next, we write in the value for "a" in our general $(x-a)$ form for the divisor (what we are dividing by). Because we generally say $(x-a)$, if the factor we are dividing by has a plus sign, the value of "a" is actually negative. Since we are dividing by x + 3, then our divisor is actually -3.

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -5 & 12 \\ \hline \end{array}$$

Now we have our synthetic division all set up and can solve using synthetic division.

Step 2: Bring the first number, 2, down below.

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -5 & 12 \\ \hline & 2 & & & \end{array}$$

Steps 3 & 4: Multiply 2 by the value to the left of the box, -3, which equals -6. Write this product below the next number in the box, 3.

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -5 & 12 \\ & & -6 & 9 & -12 \\ \hline & 2 & & & \end{array}$$

Step 5: Add 3 and -6 vertically, which equals -3. Write this below the box.

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -5 & 12 \\ & & -6 & 9 & -12 \\ \hline & 2 & -3 & & \end{array}$$

Step 6: We repeat this process to complete the table:

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -5 & 12 \\ & & -6 & 9 & -12 \\ \hline & 2 & -3 & 4 & 0 \end{array}$$

The numbers outside of the box are coefficients to the quotient. The last number, 0, represents the remainder. Then moving from right to left, the next number, 4, is the constant, the next number, -3, is the coefficient for *x*, and the final number, 2, is the coefficient for x^2 .

The solution is $2x^2-3x+4$ with a remainder of zero, which is exactly what we got using the long division process in the section above.

BIG IDEA

With synthetic division and long division of polynomials, the degree of the quotient is one less than the degree of the dividend. In the example above, the dividend $2x^3+3x^2-5x+12$ had a degree of 3. So the quotient will have a degree of 2, and we can use this to create our solution $2x^2-3x+4$.

HINT

When first creating the box, it is important that these coefficients represent terms that are written in descending order of their degree. If a polynomial is "missing a term", we must use 0 as a placeholder when writing the coefficients in the box above. This will ensure that the coefficients we get in our answer match to the proper term in the quotient. For example, x^3+5x+4 has no x^2 term, so the box would look like:

$$\begin{array}{|rrrr} 1 & 0 & 5 & 4 \\ & & & \\ \hline \end{array}$$

HINT

If there is a non-zero number at the end, then this is the remainder. To write the remainder in our solution, we must use that number as the numerator of a fraction, with the divisor $(x-a)$ as the denominator. For example, let's say we worked through the synthetic division, and our last number was a -2, instead of 0. We would write our solution as:

$$2x^2 - 3x + 4 - \frac{2}{x+3}$$

SUMMARY

When dividing polynomials, you can verify the answer, or the quotient, is correct by multiplying the quotient by the divisor to see that it equals the original dividend. Dividing polynomials **using polynomial long division** involves using the standard algorithm for long division. We can also use **synthetic division**, which uses the coefficients of the dividend polynomial and the constant term of the divisor.