The Product Property of Exponents

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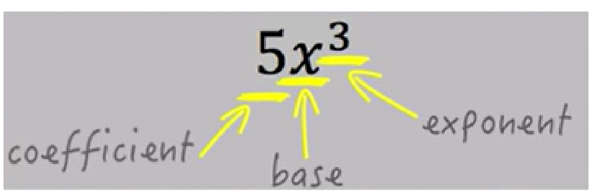
WHAT'S COVERED

This tutorial covers multiplying monomials utilizing the product property of exponents, through the definition and discussion of:

1. Monomials

Monomials are exponential expressions with non-negative integer exponents. There are three parts to a monomial: the base, the coefficient, and the exponent. The base is repeatedly multiplied by itself according to the exponent.

EXAMPLE Consider the monomial shown below:



- The base is x.
- The exponent is 3, and indicates how many times the base is used in repeated multiplication.
- The coefficient is 5, which is the number that acts as a multiplier to the base and the exponent.

HINT

It's important to note that the exponent is only being applied to the base and not the coefficient.

TERM TO KNOW

Monomial
An exponential expression with non-negative integer exponents

2. Multiplying Monomials/The Product Property of Exponents

You can multiply monomials with and without coefficients.

EXAMPLE Consider this expression without coefficients:

$(x^2)(x^4)$

To simplify, you can expand the expression using repeated multiplication. Therefore, x squared becomes x times x, and x to the fourth becomes x times x times x times x. Multiplying the x terms together provides x to the sixth.

$(x \cdot x)(x \cdot x \cdot x \cdot x)$

In the expression above, you can also determine the exponent of the product by adding the two exponents, 2 plus 4, together to provide x to the sixth.

x^6

This illustrates **the product property for exponential expressions**, which works for the product of all monomials when the bases of the monomials are the same. In general, this states that x^a times x^b is equal to x^{a+b}.

FORMULA TO KNOW

Product Property of Exponents
$(x^a)(x^b) = x^{(a+b)}$

EXAMPLE Consider the expression that involves multiplying monomials *with* coefficients:

$(-3x)(5x^6)$

You can use the commutative property of multiplication, which allows you to group your coefficients and variables together. Multiplication is commutative because you can multiply numbers in any order. Therefore, grouping your coefficients and your x terms together gives you:

$(-3 \cdot 5)(x \cdot x^6)$

Multiplying -3 times 5 equals -15, and multiplying x times x^6 is the same as x^1 times x^6. Adding your exponents using the product property for exponents gives you x^7, providing your final solution of:

$$-15x^7$$

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3. Rational Exponents

Exponents are generally integers or fractions, but they may be any number, such as a decimal. A rational exponent is an exponent that can be represented as a fraction.

The product property applies to all types of exponents, including integers and fractions. Therefore, you *add* fractions when applying the product property of exponents to rational exponents. You can easily add fractions when they have common denominators:

- First, write equivalent fractions with a common denominator.
- Next, add the numerators, leaving the denominator unchanged.
- Finally, reduce the fraction, if possible, by canceling out common factors in the numerator and denominator.

EXAMPLE Suppose you want to simplify the following expression using the product property with rational exponents.

$$(2x^{\frac{1}{2}})(4x^{\frac{5}{2}})$$

Start by multiplying your coefficients, 2 times 4. Then, group your x terms together:

$$(2 \cdot 4)(x^{\frac{1}{2}} \cdot x^{\frac{5}{2}})$$

You need to add your exponents, meaning that you need to add the fractions together. Your denominators are already the same, so you can simply add your numerators together.

$$\frac{1}{2}+\frac{5}{2}=\frac{6}{2}$$

Now, both your numerator and denominator have a common factor of 2, so you can cancel it out by dividing them both by 2, which gives you 3 over 1, or 3.

$$\frac{6}{2}=\frac{6 \div 2}{2 \div 2}=\frac{3}{1}=3$$

Bringing back in the product of your coefficients, your final expression becomes:

$8x^3$

TRY IT

Consider the following expression:

$$(-x^{\frac{3}{4}})(7x^{\frac{1}{10}})$$

Simplify this expression.

+

Start by multiplying your coefficient. The negative in front of the x is the same as a coefficient of a negative one. Therefore, you have -1 times 7, which is -7. Next, group your x terms together to multiply them:

$$(-1\cdot 7)(x^{\frac{3}{4}}\cdot x^{\frac{1}{10}}) =$$
$$(-7)(x^{\frac{3}{4}}\cdot x^{\frac{1}{10}})$$

Remember, using the product property of exponents, when multiplying your x terms, you will add the exponents—which in this case are fractions—together. When adding your fractions together, notice that the denominators are not the same. Therefore, you need to think of the least common denominator.

The least common denominator of 4 and 10 is 20. To get a denominator of 20 in the first fraction, multiply by 5 in the denominator and the numerator. To get a denominator of 20 in the second fraction, multiply by 2 in the denominator and the numerator. Now that you have a common denominator, you can add your numerators, arriving at the fraction 17/20. This fraction is fully simplified because 17 and 20 have no other common factor than 1.

$$\frac{3}{4}+\frac{1}{10}=$$
$$\frac{3\cdot 5}{4\cdot 5}+\frac{1\cdot 2}{10\cdot 2}=$$
$$\frac{15}{20}+\frac{2}{20}=$$
$$\frac{17}{20}$$

Bringing back in the product of your coefficients, then, your final expression is:

$-7x^{\frac{17}{20}}$

SUMMARY

Today you learned the definition of **monomials** and about the three parts of a monomial: the base, the coefficient, and the exponent. You also learned how to **multiply monomials**, with and without coefficients, using the **product property of exponents**, which states that the product of two monomials with the same base can be simplified by adding the original exponents. Finally, you learned about **rational exponents** and how to multiply monomials with rational exponents using the product property.

Source: This work is adapted from Sophia author Colleen Atakpu.

TERMS TO KNOW

Monomial
An exponential expression with non-negative integer exponents.

FORMULAS TO KNOW

Product Property of Exponents
$(x^a)(x^b) = x^{(a+b)}$

