The Quadratic Formula

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to solve a quadratic equation using the quadratic formula. Specifically, this lesson will cover:

1. The Quadratic Formula

The quadratic formula is used to solve quadratic equations that are written in standard form $ax^2 + bx + c$ and set equal to zero. It uses the coefficients a, *b*, and *c* that are found in the standard equation. The quadratic formula is:

FORMULA TO KNOW

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

It looks like a complete mess, doesn't it? It can be a complicated formula to work through, but it is extremely useful in finding precise solutions to quadratic equations.

BIG IDEA

It is important to note that the quadratic equation must be in the form $ax^2 + bx + c = 0$.

2. When to Use the Quadratic Formula

As we saw in a previous lesson, one of the easiest ways to solve quadratic equations is to factor the expression and use the Zero Factor Property.

EXAMPLE Find the solutions for the quadratic equation $x^2+5x+6=0$.

$x^2+5x+6=0$	Find two integers that multiply to 6 and add to 5
$(x+2)(x+3)=0$	Use the Zero Factor Property and set each factor equal to zero
$x+2=0,\ x+3=0$	Evaluate
$x=-2,\ x=-3$	Our solutions

However, consider a more complicated quadratic equation where it cannot be solved easily with factoring:

EXAMPLE Find the solutions for the quadratic equation $x^2+4x-10=0$.

We would need to find two numbers, *p* and *q*, that multiply to -10 and also add to 4. Looking at the factors of -10, we have:

p	*q*	sum: *p* + *q*
1	-10	-9
-1	10	9
2	-5	-3
-2	5	3

None of the pairs add up to 4, so this cannot be solved easily using factoring. We'll want to use a different method, such as the quadratic formula.

The quadratic expression in this example is known as a prime quadratic. Prime quadratics cannot be written in factored form, which means that factoring and solving using the Zero Factor Property is out of the question. In such cases, when we either cannot factor or are having trouble figuring out how to factor it, we can use the quadratic formula.

The quadratic formula can also tell us if there is a real solution to the quadratic equation. The expression underneath the radical is called the discriminant of the expression. Since it is underneath a square root, it must have a non-negative value to result in a real number.

- If the discriminant is non-negative (zero or greater), the quadratic has at least one real solution
- If the discriminant is negative (less than zero), the quadratic has no real solutions.

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3. Solving Equations using the Quadratic Formula

When using the quadratic formula to solve a quadratic equation, use these steps:

STEP BY STEP

1. Set the equation equal to zero: $ax^2+bx+c=0$.
2. Identify values for a, *b*, and *c*.
3. Plug these values into the quadratic formula.
4. Simplify what is underneath the radical first: evaluate b^2 and $4ac$, then find the difference. This is called the discriminant.
5. Add the square root of the discriminant to -*b* and divide by $2a$. This is one solution for *x*.
6. Subtract the square root of the discriminant from -*b* and divide by $2a$. This is the other solution for *x*.

Let's use the quadratic formula to solve the following equation:

EXAMPLE Find the solutions to the quadratic equation $2x^2+9x-5$.

At a quick glance, this doesn't seem like a quadratic that can be easily factored. Let's try using the quadratic formula instead.

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	Identify the values for a, *b*, and *c* in the equation $2x^2+9x-5$
$a=2,\ b=9,\ c=-5$	Substitute these values in the quadratic formula
$x=\dfrac{-(9)\pm\sqrt{(9)^2-4(2)(5)}}{2(2)}$	In the discriminant, square 9 and multiply 4, 2, and -5
$x=\dfrac{-9\pm\sqrt{81-(-40)}}{2(2)}$	Find the difference in the discriminant
$x=\dfrac{-9\pm\sqrt{121}}{2(2)}$	Evaluate the square root of 121
$x=\dfrac{-9\pm 11}{2(2)}$	Create two solutions, one with addition and one with subtraction
$x=\dfrac{-9-11}{2(2)},\quad x=\dfrac{-9+11}{2(2)}$	Evaluate
$x=\dfrac{-20}{4},\quad x=\dfrac{2}{4}$	Simplify
$x=-5,\quad x=0.5$	Our solutions

Let's look at a little more complicated problem.

EXAMPLE Find the solutions to the quadratic equation $x^2+6x+8=3$.

Don't jump the gun on identifying a, *b*, and *c* to use in the quadratic formula. Remember, the equation needs to be set equal to zero before we begin to find solutions. Always be sure to check that your equation is set equal to zero. If it's not, we simply add or subtract terms from both sides of the equation, until zero is on one side of the equation, and everything else is on the other. For this specific equation, simply subtract 3 from both sides to get zero on the right side.

$$x^2+6x+5=0$$

Now we can use the quadratic formula and identify the variables a, *b*, and *c*.

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	Identify the values for a, *b*, and *c* in the equation $x^2+6x+5=0$
$a=1, b=6, c=5$	Substitute these values in the quadratic formula
$x=\frac{-(6)\pm\sqrt{(6)^2-4(1)(5)}}{2(1)}$	In the discriminant, square 6 and multiply 4, 1, and 5
$x=\frac{-6\pm\sqrt{36-20}}{2(1)}$	Find the difference in the discriminant
$x=\frac{-6\pm\sqrt{16}}{2(1)}$	Evaluate the square root of 16
$x=\frac{-6\pm4}{2(1)}$	Create two solutions, one with addition and one with subtraction
$x=\frac{-6-4}{2(1)},\quad x=\frac{-6+4}{2(1)}$	Evaluate
$x=\frac{-10}{2},\quad x=\frac{-2}{2}$	Simplify
$x=-5,\quad x=-1$	Our solutions

SUMMARY

The **quadratic formula** is used to solve quadratic equations that are written in standard form and set equal to zero. It uses the coefficients a, *b*, and *c* that are found in the standard equation. Prime quadratics cannot be factored because there are no two integers that will multiply to the constant term *c* and add to the *b* coefficient. **Using the quadratic formula** can be helpful in finding the solutions to *x*. **When solving equations using the quadratic formula**, the equation needs to be set equal to zero and then use the coefficients for a, *b*, and *c*. If the value of the discriminant (*b* squared minus $4ac$) is negative, the equation has no real solution. If the value of the discriminant is non-negative, the equation has at least one real solution.

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FORMULAS TO KNOW

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$