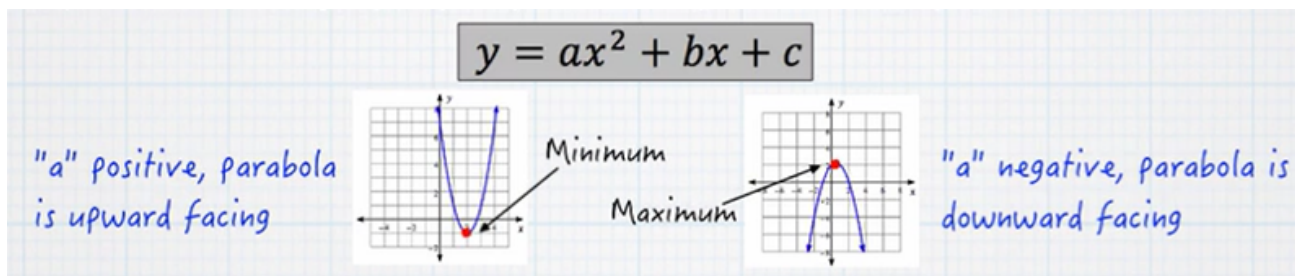# The Vertex Formula

by Sophia

1. Vertex

A parabola is the term used for a graph of a quadratic equation, which can generally be written as:

$$y = ax^2 + bx + c$$

If the coefficient a is positive, the graph is a U shape pointing up. If the coefficient a is negative, the graph is a U shape pointing down. A parabola will have a maximum point if the graph is pointing down or a minimum point if the graph is pointing up. The maximum or minimum point of a parabola is called the vertex.

The vertex provides useful information about the highest or lowest value in a quadratic relationship. These highest or lowest values are related to optimization problems, such as maximum profit or minimum cost.

2. The Vertex Formula

You can use the equation for a parabola to find the vertex without looking at the graph. When doing so, it's important to remember the following points about a parabola:

- A parabola is the graph of a quadratic equation in the form: $y = ax^2 + bx + c$.
- When the coefficient a is positive, the parabola will be upward-facing. When the coefficient a is negative, the parabola will be downward-facing.
- The vertex is the maximum or minimum point on the parabola.

The x-coordinate of the point of the vertex can be found using the following formula, in which a and b are the coefficients in your quadratic equation.

🧪 **FORMULA TO KNOW**

x-coordinate of Vertex
$$x = \frac{-b}{2a}$$

Once you have used the formula to find the x-coordinate of the vertex, you can substitute this value in for x in the equation to determine the y-coordinate of the vertex.

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3. Finding the Vertex

Suppose an Olympic diver is competing for a medal. Her dive can be modeled by the equation and graph shown below, where *x* is the time in seconds after she begins the dive and *y* is the height in feet above the water. In this example, the vertex of the parabola is a maximum point, so by finding the vertex using the vertex formula, you can determine the maximum height of her dive.
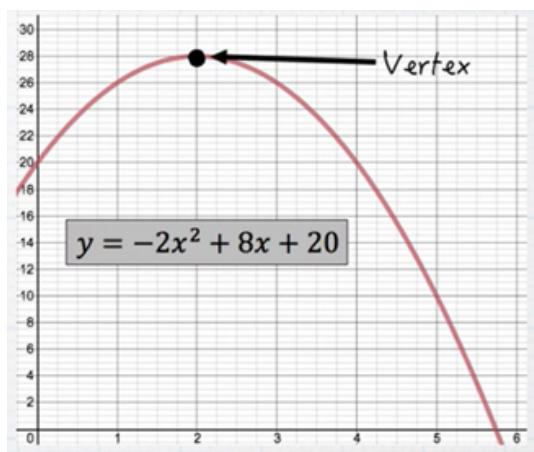
First, you take your values for *a* and *b* from your equation, and substitute them into the formula for the x coordinate of the vertex (the vertex formula).

$y = -2x^2 + 8x + 20$

In this equation, we have:

- $a = -2$
- $b = 8$

Using the formula, this gives us:

$$x = \frac{-b}{2a} = \frac{-8}{2(-2)}$$

Multiply and divide the values accordingly to provide x equals 2, so the x-coordinate of your vertex is 2.

$$x = \frac{-8}{-4} = 2$$

Next, substitute 2 into your original equation for x to find the y-value of the vertex.

$$y = -2x^2 + 8x + 20$$
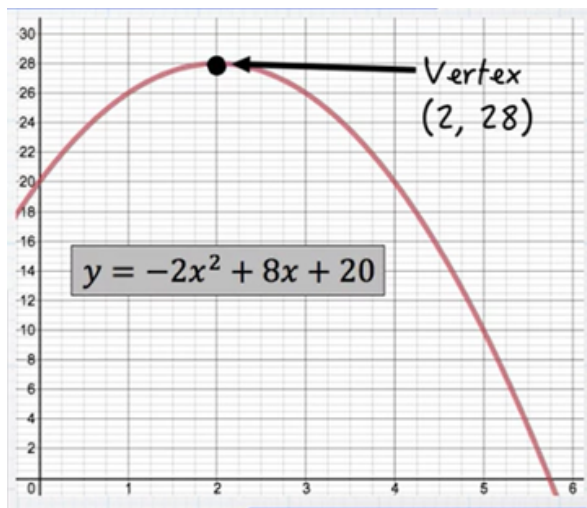$$y = -2(2)^2 + 8(2) + 20$$

Simplify with your exponent, then multiply and add, according to the order of operations.

$$y = -2(4) + 8(2) + 20$$
$$y = -8 + 16 + 20$$
$$y = 28$$

The y-coordinate of your vertex is 28, and you can see that your vertex is at the point (2, 28). This means that 2 seconds after she begins the dive, she reaches her maximum height of 28 feet above the water.
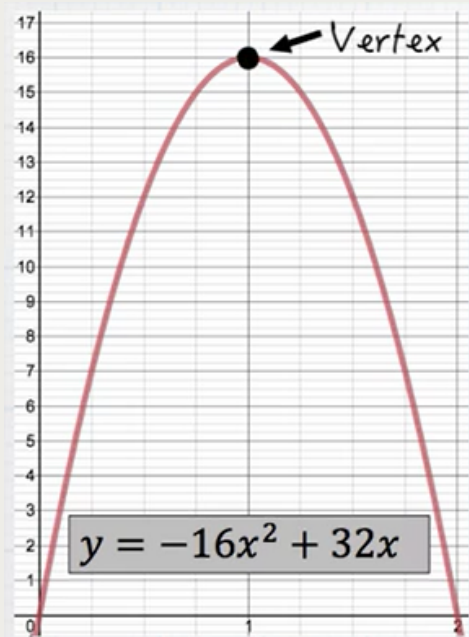

IN CONTEXT

Natasha kicks a soccer ball during a game. How can you determine the maximum height of the ball?

Well, the flight of the ball can be modeled by the equation and graph shown below, where x is the time in seconds after she kicks the ball and y is the height of the ball in feet. Note that the vertex is a

maximum point.

You can start by using your formula for the x-coordinate of the vertex and substituting your values in for *b* and *a*.

$y = -16x^2 + 32x$

In this equation, we have:
- $a = -16$
- $b = 32$

Using the formula, this gives us:

$$x = \frac{-b}{2a} = \frac{-32}{2(-16)}$$

Simplify the denominator and divide these values to provide the value of the x-coordinate of the vertex, which is 1.

$$x = \frac{-32}{-32} = 1$$

Next, substitute 1 in for x in your original equation to find the y-coordinate of the vertex.
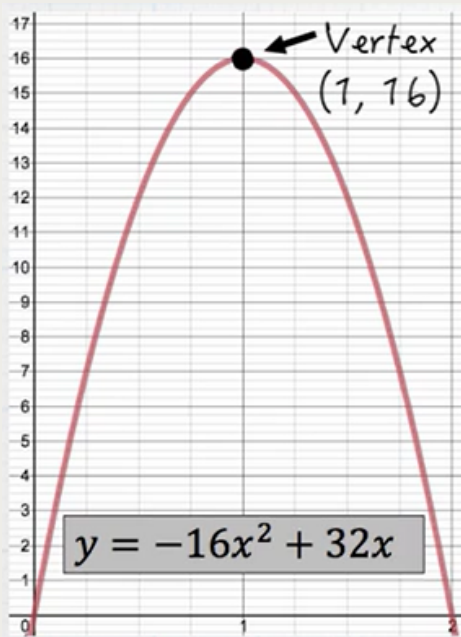
$y = -16x^2 + 32x$
$y = -16(1)^2 + 32(1)$

Simplify your exponent, multiply, and add.

$y = -16(1) + 32(1)$
$y = -16 + 32$
$y = 16$

The y-coordinate of your vertex is 16. Therefore, your vertex is at the point (1, 16). The maximum height of the ball is reached after one second and the maximum height is 16 feet.

$$y = -16x^2 + 32x$$

🗹 SUMMARY

Today you learned that the maximum or minimum point of a parabola is called the **vertex**, noting that a parabola, the graph of a quadratic equation, will have a maximum point if the graph is pointing down or a minimum point if the graph is pointing up. You also learned that you can **find the vertex** of a parabola using **the vertex formula**, by determining the x-coordinate of the point of the vertex. Once you have the x-coordinate of the vertex, you can then substitute this value in for x in the equation to find the y-coordinate of the vertex.

Source: This work is adapted from Sophia author Colleen Atakpu.

⚗ FORMULAS TO KNOW

x-coordinate of Vertex

$$x = \frac{-b}{2a}$$