Time Value of Money Calculations

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WHAT'S COVERED

In this lesson, you will calculate the effect of time and interest on financial decisions pertaining to planning, investing, and borrowing. Specifically, this lesson will cover:

1. TVM: Plan with Confidence

During your lifetime financial journey, you will be faced with many unique financial opportunities to invest, borrow, and plan for the future. Knowing how much you can afford to borrow before you apply for a loan or talk with a lender is a powerful tool. Additionally, having the ability to determine your loan payment based on the amount borrowed and interest rate charged might just protect you from being overcharged by thousands of dollars. Finally, calculating how much you should be saving for long-term goals will give you confidence in your plans or the opportunity to be agile and adjust your plans while you still have time to make changes.

In this topic, you'll learn how to perform your own time value of money (TVM) calculations, which is an essential tool for building a strong financial future.

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2. Solving TVM Problems

There are multiple ways to solve TVM problems. Most often people prefer to use a spreadsheet program like Microsoft® Excel or Google® Sheets, a financial calculator, or an app designed to solve these types of problems. These tools can help improve your productivity and they can make it easier to adjust plans when needed. However, there is value in learning how to do TVM calculations yourself either using formulas or compound interest tables. We will provide some basic steps to follow when using a calculator to solve TVM problems. However, you should consult the user manual specific to your calculator for more detailed instructions.

Technology: Skill Reflect

Are there ways you currently use programs like Excel? If so, what are they? If not, how can you begin to use technology tools like Excel in your daily life to help increase your comfort and confidence with spreadsheets?

HINT

When you see the term "annuity due," remember that the payments or deposits come at the beginning of the period. The annuity due formula accounts for this by inflating the traditional result for the calculation's rate of return. All of the other formulas use end-of-period assumptions.

2a. Future Value of a Lump Sum

Here's an example showing how to calculate a future value.

EXAMPLE Let's say that you receive $1,000 at your college graduation. If you invest the gift and earn 8% annually, how much will you have in 20 years? Let's first solve using the following formula.

$$FV = PV(1 + I)^N$$
$$FV = \$1{,}000 \times (1.08)^{20}$$
$$FV = \$1{,}000 \times 4.66096$$
$$FV = \$4{,}660.96$$

Next, let's solve the problem using a TVM table. The table below shows the Future Value of $1 table that you'll need to use. Here's how to use the table.

STEP BY STEP

1. Find the number of periods (20) in the first column.
2. Move to the right until you find the future value factor that corresponds to the 8% column: 4.66096.
3. Multiply the factor by your present value ($1,000): $1,000 x 4.6609, or $4,660.96.

You should notice two things. First, the solution is the same whether calculated with the formula or the table. Second, the future value factor in the table (4.66096) is basically the same as $(1.08)^{20}$ in the formula.

Table: Future Value of $1

Periods	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.01000	1.11000	1.12000

2	1.08160	1.10250	1.12360	1.14490	1.16640	1.18810	1.21000	1.23210	1.25440
3	1.12486	1.15763	1.19102	1.22504	1.25971	1.29503	1.33100	1.36763	1.40493
4	1.16986	1.21551	1.26248	1.31080	1.36049	1.41158	1.46410	1.51807	1.57352
5	1.21665	1.27628	1.33823	1.40255	1.46933	1.53862	1.61051	1.68506	1.76234
6	1.26532	1.34010	1.41852	1.50073	1.58687	1.67710	1.77156	1.87041	1.97382
7	1.31593	1.40710	1.50363	1.60578	1.71382	1.82804	1.94872	2.07616	2.21068
8	1.36857	1.47746	1.59385	1.71819	1.85093	1.99256	2.14359	2.30454	2.47596
9	1.42331	1.55133	1.68948	1.83846	1.99900	2.17189	2.35795	2.55804	2.77308
10	1.48024	1.62889	1.79085	1.96715	2.15892	2.36736	2.59374	2.83942	3.10585
11	1.53945	1.71034	1.89830	2.10485	2.33164	2.58043	2.85312	3.15176	3.47855
12	1.60103	1.79586	2.01220	2.25219	2.51817	2.81266	3.13843	3.49845	3.89598
13	1.66507	1.88565	2.13293	2.40985	2.71962	3.06580	3.45227	3.88328	4.36349
14	1.73168	1.97993	2.26090	2.57853	2.93719	3.34173	3.79750	4.31044	4.88711
15	1.80094	2.07893	2.39656	2.75903	3.17217	3.64248	4.17725	4.78459	5.47357
16	1.87298	2.18287	2.54035	2.95216	3.42594	3.97031	4.59497	5.31089	6.13039
17	1.94790	2.29202	2.69277	3.15882	3.70002	4.32763	5.05447	5.89509	6.86604
18	2.02582	2.40662	2.85434	3.37993	3.99602	4.71712	5.55992	6.54355	7.68997
19	2.10685	2.52695	3.02560	3.61653	4.31570	5.14166	6.11591	7.26334	8.61276
20	2.19112	2.65330	3.20714	3.86968	4.66096	5.60441	6.72750	8.06231	9.64629

2b. Present Value of a Lump Sum

Imagine you are offered the choice of taking $1,000 today or $1,200 in 5 years. Before you can make a decision, you also need to know what rate of return you can earn on your savings (sometimes called the **discount rate**). You determine that you can earn 5%. What should you do? You should use the present value of a lump sum formula to compare the $1,000 that you can receive today to the present value of $1,200 as follows:

$$PV = \frac{FV}{(1+I)^N}$$

$$PV = \frac{\$1,200}{1.05^5}$$

$$PV = \frac{\$1,200}{1.27628}$$

$$PV = \$940.23$$

As you can see, the present value of receiving $1,200 in 5 years is only $940.22. So, it’s better to take the $1,000 right now.

The table below shows the Present Value of $1 table that you can also use to solve the problem.

HINT

Note that the Present Value of $1 table is set up exactly the same as the Future Value of $1 table.

Table: Present Value of $1

Periods	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	0.96154	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286
2	0.92456	0.90703	0.89000	0.87344	0.85734	0.84168	0.82645	0.81162	0.79719
3	0.88900	0.86384	0.83962	0.81630	0.79383	0.77218	0.75131	0.73119	0.71178
4	0.85480	0.82270	0.79209	0.76290	0.73503	0.70843	0.68301	0.65873	0.63552
5	0.82193	0.78353	0.74726	0.71299	0.68058	0.64993	0.62092	0.59345	0.56743
6	0.79031	0.74622	0.70496	0.66634	0.63017	0.59627	0.56447	0.53464	0.50663
7	0.75992	0.71068	0.66506	0.62275	0.58349	0.54703	0.51316	0.48166	0.45235
8	0.73069	0.67684	0.62741	0.58201	0.54027	0.50187	0.46651	0.43393	0.40388
9	0.70259	0.64461	0.59190	0.54393	0.50025	0.46043	0.42410	0.39092	0.36061
10	0.67556	0.61391	0.55839	0.50835	0.46319	0.42241	0.38554	0.35218	0.32197
11	0.64958	0.58468	0.52679	0.47509	0.42888	0.38753	0.35049	0.31728	0.28748
12	0.62460	0.55684	0.49697	0.44401	0.39711	0.35553	0.31863	0.28584	0.25668
13	0.60057	0.53032	0.46884	0.41496	0.36770	0.32618	0.28966	0.25751	0.22917
14	0.57748	0.50507	0.44230	0.38782	0.34046	0.29925	0.26333	0.23199	0.20462
15	0.55526	0.48102	0.41727	0.36245	0.31524	0.27454	0.23939	0.20900	0.18270
16	0.53391	0.45811	0.39365	0.33873	0.29189	0.25187	0.21763	0.18829	0.16312
17	0.51337	0.43630	0.37136	0.31657	0.27027	0.23107	0.19784	0.16963	0.14564
18	0.49363	0.41552	0.35034	0.29586	0.25025	0.21199	0.17986	0.15282	0.13004
19	0.47464	0.39573	0.33051	0.27651	0.23171	0.19449	0.16351	0.13768	0.11611
20	0.45639	0.37689	0.31180	0.25842	0.21455	0.17843	0.14864	0.12403	0.10367

STEP BY STEP

1. Find the number of periods in column 1, which is 5.
2. Go to the right until you find the present value factor in the 5% column: 0.78353.
3. Multiply $1,200 by 0.78353 and you will get $940.24 (which is very close to the formula calculation).

TERM TO KNOW

Discount Rate

Rate of return you can earn on your savings.

2c. Future Value of an Annuity

By now, you might be asking how you can determine how much money you'll have in the future if you save money every year instead of starting with a lump sum. When you start saving money on a regular basis, this is called an *annuity*. In TVM lingo, the amount saved or paid each period is referred to as a *payment (PMT)*. Payments are different than present or future values because, as the name implies, a payment happens more than once.

EXAMPLE How much, for example, will you accumulate in 20 years if you start saving $1,000 *every year* (each payment is the same amount) and earn 9% on your savings? The following formula can be used to answer this question:

$$FV\ A = \frac{PMT}{I}(1+I)^{N} - 1$$
$$FV\ A = \frac{\$1{,}000}{0.09} \times (1+0.09)^{20} - 1$$
$$FV\ A = \$11{,}111.11 \times 4.60441$$
$$FV\ A = \$51{,}160.11$$

You can also solve this problem with a TVM table. The table below shows the Future Value of an Annuity of $1 table that can be used to solve the problem as follows:

STEP BY STEP

1. Just like the other table examples, start by finding the number of payments (20) in the first column.
2. Go to the right until you find the future value factor corresponding to the 9% column: 51.16012.
3. Multiply $1,000 by 51.16012. This gives you a future value of $51,160.12, which is nearly the same as what you estimated using the formula.

Table: Future Value of Annuity for $1 at the End of Each Period

Per	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.04000	2.05000	2.06000	2.07000	2.08000	2.09000	2.10000	2.11000	2.12000
3	3.12160	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000	3.34210	3.37440
4	4.24646	4.31013	4.37462	4.43994	4.50611	4.57313	4.64100	4.70973	4.77933
5	5.41632	5.52563	5.63709	5.75074	5.86660	5.98471	6.10510	6.22780	6.35285
6	6.63298	6.80191	6.97532	7.15329	7.33592	7.52334	7.71561	7.91286	8.11519

7	7.89829	8.14201	8.39384	8.65402	8.92280	9.20044	9.48717	9.78327	10.08901
8	9.21423	9.54911	9.89747	10.25980	10.63663	11.02847	11.43589	11.85943	12.29969
9	10.58280	11.02656	11.49132	11.97799	12.48756	13.02104	13.57948	14.16397	14.77566
10	12.00611	12.57789	13.18079	13.81645	14.48656	15.19293	15.93743	16.72201	17.54874
11	13.48635	14.20679	14.97164	15.78360	16.64549	17.56029	18.53117	19.56143	20.65458
12	15.02581	15.91713	16.86994	17.88845	18.97713	20.14072	21.38428	22.71319	24.13313
13	16.62684	17.71298	18.88214	20.14064	21.49530	22.95339	24.52271	26.21164	28.02911
14	18.29191	19.59863	21.01507	22.55049	24.21492	26.01919	27.97498	30.09492	32.39260
15	20.02359	21.57856	23.27597	25.12902	27.15211	29.36092	31.77248	34.40536	37.27972
16	21.82453	23.65749	25.67253	27.88805	30.32428	33.00340	35.94973	39.18995	42.75328
17	23.69751	25.84037	28.21288	30.84021	33.75023	36.97351	40.54470	44.50084	48.88367
18	25.64541	28.13238	30.90565	33.99903	37.45024	41.30134	45.59917	50.39593	55.74972
19	27.67123	30.53900	33.75999	37.37896	41.44626	46.01846	51.15909	56.93949	63.43968
20	29.77808	33.06595	36.78559	40.99549	45.76196	51.16012	57.27500	64.20283	72.05244

THINK ABOUT IT

The present value of an annuity is another TVM calculation that's useful when planning your finances. What do you think are similarities and differences between the calculations for the present value of an *annuity* and the present value of a *lump sum*?

Note: calculations for present value of an annuity are beyond the scope of this tutorial.

2d. Payments

There are times you'll need to calculate what is called an **amortized payment** – a payment of the same amount for a set number of months or years – such as for a car loan or mortgage. To do so, you should use the following formula:

FORMULA TO KNOW

$$\text{Monthly Payment} = PV\left[\frac{I \times (1+I)^N}{(1+I)^N - 1}\right]$$

In the formula, *PV* is the amount borrowed, *I* = interest rate, and *N* = number of payments.

EXAMPLE For example, say that Max wants to borrow $250,000 to purchase a home. He can get a 30-year loan at a 6% interest rate. To help Max determine if he can afford this loan, you first need to convert the years to months and the yearly rate of interest to monthly interest because Max will be making monthly payments.

- Number of periods (N) = 30 years × 12 months = 360

- Monthly interest rate (I) = 6% / 12 = 0.5% = 0.005

You can now use the following formula to determine how much Max's *monthly* payment will be (that is, you can calculate his principal and interest payment).

$$\text{Monthly Payment} = PV\left[\frac{I \times (1+I)^N}{(1+I)^N - 1}\right]$$

$$\text{Monthly Payment} = \$250{,}000\left[\frac{0.005 \times (1+0.005)^{360}}{(1+0.005)^{360} - 1}\right]$$

$$\text{Monthly Payment} = \$250{,}000\left[\frac{0.005 \times 6.0226}{6.0226 - 1}\right]$$

$$\text{Monthly Payment} = \$1{,}498.88$$

If you round the answer, Max will need to pay approximately $1,500 per month for the next 30 years to pay off the home mortgage loan.

You can also solve this problem using an amortization schedule. The factors shown in the table below show the monthly dollar payment needed to pay off a $1,000 loan.

Table: Amortization Schedule for Monthly Payments for Every $1,000 Borrowed

Years →	5	10	15	20	25	30
3.00%	$17.97	$9.66	$6.91	$5.55	$4.74	$4.22
3.50%	$16.67	$8.33	$5.56	$4.17	$3.33	$2.78
4.00%	$18.42	$10.12	$7.40	$6.06	$5.28	$4.77
4.50%	$18.64	$10.36	$7.65	$6.33	$5.56	$5.07
5.00%	$18.87	$10.61	$7.91	$6.60	$5.85	$5.37
5.50%	$19.10	$10.85	$ 8.17	$6.88	$6.14	$5.68
6.00%	$19.33	$11.10	$8.44	$7.16	$6.44	$6.00
6.50%	$19.57	$11.35	$8.71	$7.46	$6.75	$6.32
7.00%	$19.80	$11.61	$8.99	$7.75	$7.07	$6.65
7.50%	$20.04	$11.87	$9.27	$8.06	$7.39	$6.99
8.00%	$20.28	$12.13	$9.56	$8.36	$7.72	$7.34
8.50%	$20.52	$12.40	$9.85	$8.68	$8.05	$7.69
9.00%	$20.76	$12.67	$10.14	$9.00	$8.39	$8.05
9.50%	$21.00	$12.94	$10.44	$9.32	$8.74	$8.41
10.00%	$21.25	$13.22	$10.75	$9.65	$9.09	$8.78

To estimate the monthly payment needed by Max:

STEP BY STEP

1. Find the interest rate of the loan in the first column.
2. Go to the right until you find the amortization factor matching 30 years: $6.00 (Max will pay $6.00 for every $1,000 borrowed).
3. Divide the loan amount by $1,000: $(\$250{,}000 / \$1{,}000) = \$250$ (you need to do this to keep the factors consistent with the loan amount).
4. Multiply 250 by $6.00 to get $1,500, which is a very close approximation of the actual amount of the loan payment.

TERM TO KNOW

Amortized Payment
A payment of the same amount for a set number of months or years, such as for a car loan or mortgage.

SUMMARY

When you can mathematically **solve time value of money (TVM) problems**, it allows you to **plan your finances with confidence** when you're saving, investing, and borrowing. Several types of TVM calculations were covered in this tutorial:

- **Future Value of a Lump Sum**
- **Present Value of a Lump Sum**
- **Future Value of an Annuity**
- **Payment**

Using technology, like spreadsheets, for TVM calculations can help you increase productivity and gain the ability to be agile when needed.

TERMS TO KNOW

Amortized Payment
A payment of the same amount for a set number of months or years, such as for a car loan or mortgage.

Discount Rate

Rate of return you can earn on your savings.