Using Factoring in Rational Expressions

by Sophia

WHAT'S COVERED

This tutorial covers using factors in rational expressions, through the definition and discussion of:

1. Greatest Common Factor

Terms in a polynomial may have common factors, which are numbers that divide each term in the polynomial. These common terms can be factored out to simplify the polynomial. The greatest common factor is the product of all common factors in a polynomial.

DID YOU KNOW

Factoring out common factors is especially useful when simplifying algebraic fractions or rational expressions.

2. Rational Expressions

A rational expression is a fraction whose numerator and denominator are both polynomials. Rational expressions can be simplified in a similar way to numeric fractions:

- First, identify common factors in the numerator and denominator.
- Second, cancel out any common factors.

3. Factoring Rational Expressions

EXAMPLE Suppose you want to factor the following rational expression:

$$\frac{x^2+4x}{x^2-12x}$$

First, you'll want to identify any common factors in the numerator. The terms in the numerator both have an x; therefore, you can factor it out. After taking out an x, the remaining factors go inside the parentheses.

$$\frac{x^2+4x}{x^2-12x} = \frac{x(x+4)}{}$$

Next, identify any common factors in the denominator. The terms in the denominator also both have an x, so you can factor it out. After taking out an x, place the remaining factors inside the parentheses.

$$\frac{x^2+4x}{x^2-12x} = \frac{x(x+4)}{x(x-12)}$$

Now you can see that the numerator and denominator both have a common factor of x. You can cancel out these x terms, because x over x is equal to 1. The terms remaining in the numerator and denominator cannot be canceled out, because they are separated by addition or subtraction.

$$\frac{x^2+4x}{x^2-12x} = \frac{\cancel{x}(x+4)}{\cancel{x}(x-12)} = \frac{x+4}{x-12}$$

BIG IDEA

Only terms separated by multiplication are canceled out by the division operation of a fraction.

TRY IT

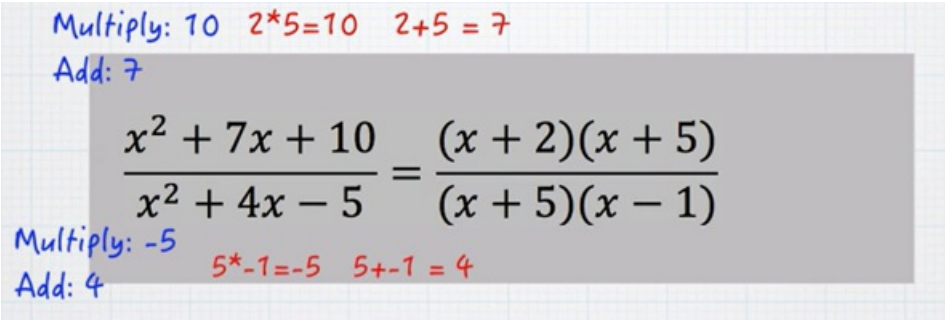
Consider the following rational expression:

$$\frac{x^2+7x+10}{x^2+4x-5}$$

Use what you've learned about common factors to simplify this rational expression. +

Did you notice that these are both quadratic expressions in the numerator and the denominator? Therefore, you may be able to write these expressions in factored form.

To factor, you need to find two numbers that multiply to the constant term but add to the coefficient of the x term. In the numerator, you need to find two numbers that multiply to 10 and add to 7. In the denominator, you need to find two numbers that multiply to -5 and add to 4. These number pairs are shown below, allowing you to write the expression in factored form.



Now, you can see that you have a common factor in the numerator and denominator of x plus 5. You

can cancel out these x plus 5 factors, because x plus 5 over x plus 5 is equal to 1.

$$\frac{x^2+7x+10}{x^2+4x-5}=\frac{(x+2)\cancel{(x+5)}}{\cancel{(x+5)}(x-1)}=\frac{x+2}{x-1}$$

SUMMARY

Today you reviewed the concept of **greatest common factor**. You learned that terms in a polynomial may have common factors, which are numbers that divide each term in the polynomial. These common terms can be factored out to simplify the polynomial. You also learned that a **rational expression** is a fraction whose numerator and denominator are polynomials, and that you can**factor rational expressions** in the same manner as numeric fractions. Lastly, you learned that it's important to remember that when canceling out terms in the numerator and denominator, only terms separated by multiplication are canceled out by the division operation of a fraction.

Source: This work is adapted from Sophia author Colleen Atakpu.