Using Quadratic Equations to Represent Motion

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WHAT'S COVERED

In this lesson, you will learn how to solve a quadratic equation that represents the path of an object in motion. Specifically, this lesson will cover:

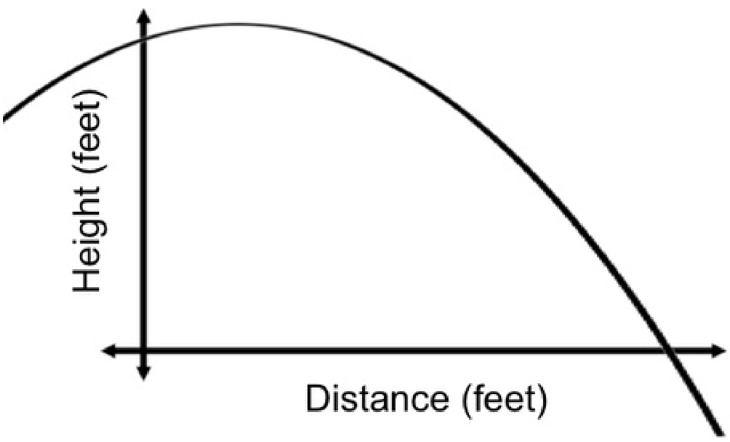
1. The Path of an Object in Motion

Quadratic equations can be used to represent the path of objects as they go up in the air and come back down due to the force of gravity.

EXAMPLE A volleyball is being shot over a net, and at some distance from the net, it hits the ground. This path can be described by the following equation:

$$y = -0.05x^2 + 0.3x + 8$$

In this equation, y equals the height of the ball, and x is the distance from the net. The graph below shows this relationship:



We can imagine the x-axis of the graph as the ground, the y-axis of the graph as the volleyball net, and the curve itself is the path of the volley.

We would like to know at what distance the volleyball hits the ground. To answer this question, we need to find the value of *x* when *y* equals zero. There are a couple of methods for solving quadratic equations set equal to zero. One method is to factor the equation, to get an equation in the form $0=(x+a)(x+b)$. Another method is to use the quadratic formula, which uses the coefficients of the equation in standard form (a, *b*, and *c*). We will use the quadratic formula to solve this equation, because by the looks of the coefficients in the equation, factoring is either going to be difficult or impossible.

First, identify the coefficients a, *b*, and *c*.

$y=-0.05x^2+0.3x+8$	Identify coefficients a, *b*, and *c*
$a=-0.05,\ b=0.3,\ c=8$	Plug coefficients into quadratic formula

Now we can use the quadratic formula to find the solutions.

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	Plug $a=-0.05,\ b=0.3,\ c=8$ into quadratic formula
$x=\dfrac{-0.3\pm\sqrt{0.3^2-4(-0.05)(8)}}{2(-0.05)}$	Square 0.3 and multiply 4, -0.5, and 8
$x=\dfrac{-0.3\pm\sqrt{0.09-(-1.6)}}{2(-0.05)}$	Evaluate radical
$x=\dfrac{-0.3\pm\sqrt{1.69}}{2(-0.05)}$	Simplify radical
$x=\dfrac{-0.3\pm1.3}{2(-0.05)}$	Evaluate denominator
$x=\dfrac{-0.3\pm1.3}{-0.1}$	Create two equations, one with addition and one with subtraction
$x=\dfrac{-0.3-1.3}{-0.1},\ x=\dfrac{-0.3+1.3}{-0.1}$	Simplify numerator in both fractions
$x=\dfrac{-1.6}{-0.1},\ x=\dfrac{1}{-0.1}$	Evaluate both fractions
$x=16,\ x=-10$	Our solutions

We have two solutions here, one which is positive and one which is negative. Looking back at our diagram, we see that taking the negative answer doesn't make sense in this context. For this reason, we'll only take the positive value, which describes the distance from the net where the volleyball hits the ground.

We can say that the volleyball will hit the ground 16 feet from the net.

HINT

Always be sure to keep the context of the problem in mind. In the example above, we did not accept the negative solution, because negative distance didn't make sense in the context.

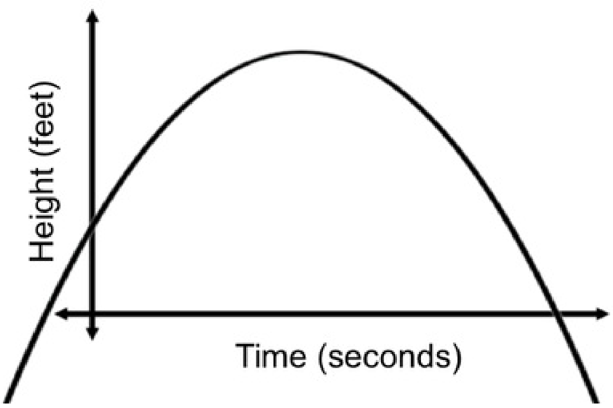
2. Calculating Maximum Height of an Object in Motion

We can also use quadratic equations to calculate the maximum height of an object that is in motion.

↪ EXAMPLE Think about a volleyball being shot in the air in a vertical motion, once again going up, and eventually coming back down to the ground. This relationship can be shown with the following quadratic equation:

$$y = -16x^2 + 80x + 6$$

In this equation, y equals the height of the ball, and x is time. The graph below shows this relationship between the height of the ball in relationship to time:



We see that the x-axis represents time, measured in seconds, and the y-axis represents height, measured in feet.

The coefficients in this equation have important meanings within the context. $-16x^2$ comes from the force of gravity on Earth, $80x$ represents the velocity of the volleyball (in this case, it is 80 ft/s), and 6 represents the height of the volleyball player hitting the ball.

We can use this quadratic equation to answer questions about the maximum height that the ball will reach before it begins to come back down to the ground. Looking back at our graph, we notice that the maximum height of the volleyball is at the vertex of the parabola. How can we algebraically determine the coordinates of a vertex?

We use the following equation to find the x-coordinate of the vertex (which is also the equation to the axis of symmetry of the parabola):

$x = -\frac{b}{2a}$, where a and b come from coefficients of the equation in standard form

First, identify the coefficients a and b in the original equation:

$y = -16x^2 + 80x + 6$	Identify coefficients a, b, and c
$a = -16, b = 80, c = 6$	Plug into formula for x-coordinate of vertex

Now we can find the x-coordinate of the vertex.

$x = -\frac{b}{2a}$	Plug $a = -16, b = 80, c = 6$ into formula
$x = -\frac{80}{2(-16)}$	Evaluate denominator
$x = -\frac{80}{-32}$	Divide 80 by -32
$x = -(-2.5)$	Simplify
$x = 2.5$	x-coordinate of vertex

Our solved value for x represents the time at which the ball will reach the maximum height. We would like to calculate the height. To do this, we must plug this value of x back into the equation and find the associated y-value.

$y = -16x^2 + 80x + 6$	Plug 2.5 in for x
$y = -16(2.5)^2 + 80(2.5) + 6$	Evaluate
$y = -100 + 200 + 6$	Simplify
$y = 106$	Our solution

This means that after 2.5 seconds, the volleyball will reach a maximum height of 106 feet.

SUMMARY

A quadratic equation can be used to model **the path of an object in motion**, rising and falling due to gravity. If x represents time and y represents height, the quadratic equation and/or its graph can be used to answer questions about the object's path such as the time to travel a certain horizontal distance, or the height of the object at a certain time. We can use this to **calculate maximum height of an object in motion** by finding the vertex.