

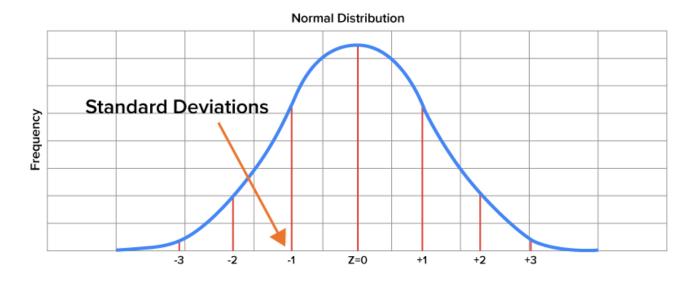
Using Z-Scores to Find a Probability

by Sophia

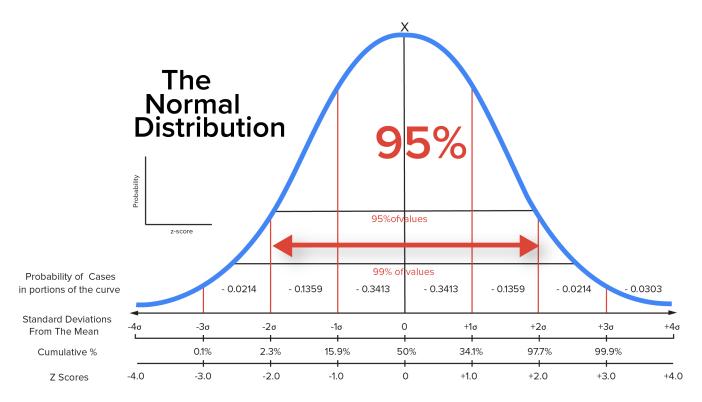
≔	WHAT'S COVERED
Tł	his lesson discusses using z-scores to find a probability. By the end of this lesson, you will be able to
us	se a z-score probability table to determine the likelihood of a range of values of a normally distributed
Va	ariable occurring. This lesson covers:
	1. Normal Distribution and Z-Scores
	2. Z-Tables
	3. Using Z-Tables to Find Probability
	3a. Higher
	3b. Range
	3c. Lower
	4. Using Z-Tables to Find Values

1. Normal Distribution and Z-Scores

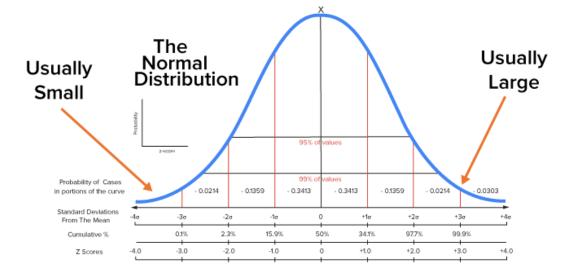
The graph you see here is that of a standard, normally distributed variable, otherwise known as a z-distribution. Because they allow you to know the position of a value in a normal distribution, z-scores are important. When you know a z-score associated with the value of a variable, you are able to determine the likelihood that an event will take place. Simply having a solid understanding of z-scores and how they are associated with probability allows you to easily determine the likelihood of an event occurring.



Approximately 68% of values in a normally distributed variable fall between a z-score of -1 and 1. Ninety-five percent of values fall between a value of -2 and 2; 99.7% of values fall between a z-score of -3 and 3. If you were to think of likely values in a normal distribution, they would fall in the 95% between a z-score of -2 and 2.



If a value has a large positive z-score, you would consider it to be unusually large, and not very likely to occur. Along the same line of thought, if a value has a large negative z-score, you would consider it to be unusually small, and not very likely to occur either.



Suppose you were to take a look at the mean area of all homes in the United States in terms of square feet. Say that the mean area of a home in the United States is 2,400 square feet, with a standard deviation of 400 square feet. The table below shows multiple z-scores.

Value	Z-Score	Graph
A home with area of 1,450 square feet	$z\text{-score} = \frac{1,450 - 2,400}{400} = -2.375$	Normal Distribution
A home with area of 2,300 square feet	$z\text{-score} = \frac{2,300 - 2,400}{400} = -0.25$	Normal Distribution
A home with area of 4,080 square feet	$z\text{-score} = \frac{4,080 - 2,400}{400} = 4.20$	Normal Distribution

🕸 THINK ABOUT IT

Of these three z-scores, which are likely to occur?

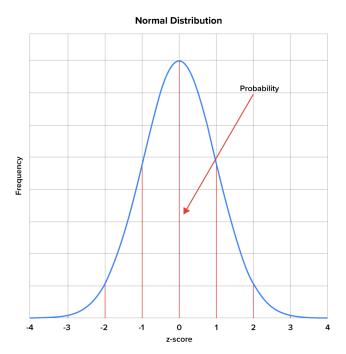
The 2,300-square-foot home is likely to occur in a sample of homes drawn from a normally distributed population. However, the 1,450-square-foot home and the 4,080-foot-square home would be very unlikely because they don't fall between a z-score of -2 and 2.

2. Z-Tables

Since the z-distribution is a standard normal distribution, z-scores and probabilities that are associated with a specific z-score can be represented on standard tables. This being the case, you can use a table to find the areas under a z-distribution curve.

When viewing a normal distribution graph, the area under the curve is associated with the probabilities of a value occurring. Knowing the area corresponding to a zscore tells us about the probability of an event taking place.

Notice the organization of the z-tables below. The negative **z-table** starts out at the left of the distribution (-3) and shows increasing values of z up to the point of z being equal to 0. The positive z-table begins at the center of the z-distribution, where z is equal to 0 and represents the mean of the distribution.



z-table for negative z-values

z-table	ior negativ	e z-values									
z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
-3	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	0.0013
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	0.0026
-2.6		0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0027	0.0036	0.0035
-2.5		0.0060	0.0059	0.0043	0.0055	0.0054	0.0055	0.0051	0.0049	0.0048	0.0035
-2.5		0.0080	0.0078	0.0075	0.0073	0.0071	0.0052	0.0051	0.0049	0.0048	0.0047
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084	0.0082
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110	0.0107
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143	0.0139
-2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183	0.0179
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233	0.0228
-1.8		0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294	0.0287
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367	0.0359
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455	0.0446
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559	0.0548
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681	0.0668
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823	0.0808
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985	0.0968
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170	0.1151
-1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379	0.1357
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611	0.1587
-0.8		0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867	0.1841
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148	0.2119
-0.6		0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451	0.2420
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2405	0.2451	0.2743
-0.5	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121	0.3085
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483	0.3446
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859	0.3821
-0.1		0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247	0.4207
0		0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641	0.4602
	for positive										
z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0	0.5000				0 54.00		0 5000				0 5000
		0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.5398
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.5793
0.2	0.5398 0.5793	0.5438 0.5832	0.5478 0.5871	0.5517 0.5910	0.5557 0.5948	0.5596 0.5987	0.5636 0.6026	0.5675 0.6064	0.5714 0.6103	0.5753 0.6141	0.5793 0.6179
0.2 0.3	0.5398 0.5793 0.6179	0.5438 0.5832 0.6217	0.5478 0.5871 0.6255	0.5517 0.5910 0.6293	0.5557 0.5948 0.6331	0.5596 0.5987 0.6368	0.5636 0.6026 0.6406	0.5675 0.6064 0.6443	0.5714 0.6103 0.6480	0.5753 0.6141 0.6517	0.5793 0.6179 0.6554
0.2 0.3 0.4	0.5398 0.5793 0.6179 0.6554	0.5438 0.5832 0.6217 0.6591	0.5478 0.5871 0.6255 0.6628	0.5517 0.5910 0.6293 0.6664	0.5557 0.5948 0.6331 0.6700	0.5596 0.5987 0.6368 0.6736	0.5636 0.6026 0.6406 0.6772	0.5675 0.6064 0.6443 0.6808	0.5714 0.6103 0.6480 0.6844	0.5753 0.6141 0.6517 0.6879	0.5793 0.6179 0.6554 0.6915
0.2 0.3 0.4 0.5	0.5398 0.5793 0.6179 0.6554 0.6915	0.5438 0.5832 0.6217 0.6591 0.6950	0.5478 0.5871 0.6255 0.6628 0.6985	0.5517 0.5910 0.6293 0.6664 0.7019	0.5557 0.5948 0.6331 0.6700 0.7054	0.5596 0.5987 0.6368 0.6736 0.7088	0.5636 0.6026 0.6406 0.6772 0.7123	0.5675 0.6064 0.6443 0.6808 0.7157	0.5714 0.6103 0.6480 0.6844 0.7190	0.5753 0.6141 0.6517 0.6879 0.7224	0.5793 0.6179 0.6554 0.6915 0.7257
0.2 0.3 0.4	0.5398 0.5793 0.6179 0.6554	0.5438 0.5832 0.6217 0.6591	0.5478 0.5871 0.6255 0.6628	0.5517 0.5910 0.6293 0.6664	0.5557 0.5948 0.6331 0.6700	0.5596 0.5987 0.6368 0.6736	0.5636 0.6026 0.6406 0.6772	0.5675 0.6064 0.6443 0.6808	0.5714 0.6103 0.6480 0.6844	0.5753 0.6141 0.6517 0.6879	0.5793 0.6179 0.6554 0.6915
0.2 0.3 0.4 0.5	0.5398 0.5793 0.6179 0.6554 0.6915	0.5438 0.5832 0.6217 0.6591 0.6950	0.5478 0.5871 0.6255 0.6628 0.6985	0.5517 0.5910 0.6293 0.6664 0.7019	0.5557 0.5948 0.6331 0.6700 0.7054	0.5596 0.5987 0.6368 0.6736 0.7088	0.5636 0.6026 0.6406 0.6772 0.7123	0.5675 0.6064 0.6443 0.6808 0.7157	0.5714 0.6103 0.6480 0.6844 0.7190	0.5753 0.6141 0.6517 0.6879 0.7224	0.5793 0.6179 0.6554 0.6915 0.7257
0.2 0.3 0.4 0.5 0.6	0.5398 0.5793 0.6179 0.6554 0.6915 0.7257	0.5438 0.5832 0.6217 0.6591 0.6950 0.7291	0.5478 0.5871 0.6255 0.6628 0.6985 0.7324	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389	0.5596 0.5987 0.6368 0.6736 0.7088 0.7422	0.5636 0.6026 0.6406 0.6772 0.7123 0.7454	0.5675 0.6064 0.6443 0.6808 0.7157 0.7486	0.5714 0.6103 0.6480 0.6844 0.7190 0.7517	0.5753 0.6141 0.6517 0.6879 0.7224 0.7549	0.5793 0.6179 0.6554 0.6915 0.7257 0.7580
0.2 0.3 0.4 0.5 0.6 0.7	0.5398 0.5793 0.6179 0.6554 0.6915 0.7257 0.7580	0.5438 0.5832 0.6217 0.6591 0.6950 0.7291 0.7611	0.5478 0.5871 0.6255 0.6628 0.6985 0.7324 0.7642	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704	0.5596 0.5987 0.6368 0.6736 0.7088 0.7422 0.7734	0.5636 0.6026 0.6406 0.6772 0.7123 0.7454 0.7764	0.5675 0.6064 0.6443 0.6808 0.7157 0.7486 0.7794	0.5714 0.6103 0.6480 0.6844 0.7190 0.7517 0.7823	0.5753 0.6141 0.6517 0.6879 0.7224 0.7549 0.7852	0.5793 0.6179 0.6554 0.6915 0.7257 0.7580 0.7881
0.2 0.3 0.4 0.5 0.6 0.7 0.8	0.5398 0.5793 0.6179 0.6554 0.6915 0.7257 0.7580 0.7881	0.5438 0.5832 0.6217 0.6591 0.6950 0.7291 0.7611 0.7910	0.5478 0.5871 0.6255 0.6628 0.6985 0.7324 0.7642 0.7939	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995	0.5596 0.5987 0.6368 0.6736 0.7088 0.7422 0.7734 0.8023	0.5636 0.6026 0.6406 0.6772 0.7123 0.7454 0.7764 0.8051	0.5675 0.6064 0.6443 0.6808 0.7157 0.7486 0.7794 0.8078	0.5714 0.6103 0.6480 0.6844 0.7190 0.7517 0.7823 0.8106	0.5753 0.6141 0.6517 0.6879 0.7224 0.7549 0.7852 0.8133	0.5793 0.6179 0.6554 0.6915 0.7257 0.7580 0.7881 0.8159
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9	0.5398 0.5793 0.6179 0.6554 0.6915 0.7257 0.7580 0.7881 0.8159	0.5438 0.5832 0.6217 0.6591 0.6950 0.7291 0.7611 0.7910 0.8186	0.5478 0.5871 0.6255 0.6628 0.6985 0.7324 0.7642 0.7939 0.8212	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264	0.5596 0.5987 0.6368 0.6736 0.7088 0.7422 0.7734 0.8023 0.8289	0.5636 0.6026 0.6406 0.6772 0.7123 0.7454 0.7764 0.8051 0.8315	0.5675 0.6064 0.6443 0.6808 0.7157 0.7486 0.7794 0.8078 0.8340	0.5714 0.6103 0.6480 0.6844 0.7190 0.7517 0.7823 0.8106 0.8365	0.5753 0.6141 0.6517 0.6879 0.7224 0.7549 0.7852 0.8133 0.8389	0.5793 0.6179 0.6554 0.6915 0.7257 0.7580 0.7881 0.8159 0.8413
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1	0.5398 0.5793 0.6179 0.6554 0.6915 0.7257 0.7580 0.7881 0.8159 0.8413	0.5438 0.5832 0.6217 0.6591 0.6950 0.7291 0.7611 0.7910 0.8186 0.8438	0.5478 0.5871 0.6255 0.6628 0.6985 0.7324 0.7642 0.7642 0.7939 0.8212 0.8461	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508	0.5596 0.5987 0.6368 0.6736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531	0.5636 0.6026 0.6406 0.772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8554	0.5675 0.6064 0.6443 0.6808 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577	0.5714 0.6103 0.6480 0.6844 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599	0.5753 0.6141 0.6517 0.6879 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621	0.5793 0.6179 0.6554 0.6915 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1	0.5398 0.5793 0.6179 0.6554 0.6915 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643	0.5438 0.5832 0.6217 0.6591 0.6950 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665	0.5478 0.5871 0.6255 0.6628 0.6985 0.7324 0.7642 0.7939 0.8212 0.8461 0.8686	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7673 0.8238 0.8485 0.8708	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729	0.5596 0.5987 0.6368 0.6736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749	0.5636 0.6026 0.6406 0.772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8554 0.8770	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790	0.5714 0.6103 0.6480 0.6844 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810	0.5753 0.6141 0.6517 0.6879 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2	0.5398 0.5793 0.6179 0.6554 0.6915 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849 0.9032	0.5438 0.5832 0.6217 0.6591 0.6950 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.8869 0.9049	0.5478 0.5871 0.6255 0.6628 0.6985 0.7324 0.7642 0.7939 0.8212 0.8461 0.8686 0.8888	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7673 0.7967 0.8238 0.8485 0.8485 0.8708 0.8907	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925	0.5596 0.5987 0.6368 0.6736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944	0.5636 0.6026 0.6406 0.772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8554 0.8770 0.8962 0.9131	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8340 0.8577 0.8790 0.8980 0.9147	0.5714 0.6103 0.6480 0.6844 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997	0.5753 0.6141 0.6517 0.6879 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849 0.9032
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4	0.5398 0.5793 0.6179 0.6554 0.6915 0.7257 0.7580 0.7881 0.8159 0.8413 0.8443 0.8849 0.8849 0.9032 0.9032	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.8869 0.9049 0.9207	0.5478 0.5871 0.6255 0.6628 0.7324 0.7642 0.7642 0.7939 0.8212 0.8461 0.8686 0.8888 0.9066 0.9222	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485 0.8485 0.8708 0.8907 0.9082 0.9236	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251	0.5596 0.5987 0.6368 0.6736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265	0.5636 0.6026 0.6406 0.772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997 0.9162 0.9306	0.5753 0.6141 0.6517 0.6879 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.8643 0.9032 0.9192 0.9192
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5	0.5398 0.6793 0.6554 0.6915 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.8849 0.9032 0.9192 0.9192	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.9049 0.9049 0.9207 0.9345	0.5478 0.6255 0.6628 0.7324 0.7642 0.7642 0.7939 0.8212 0.8461 0.8888 0.9066 0.9222 0.9357	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7673 0.7967 0.8238 0.8485 0.8485 0.8708 0.8907 0.9082 0.9082 0.9236 0.9370	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251 0.9382	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394	0.5636 0.6026 0.6406 0.772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997 0.9162 0.9306 0.9429	0.5753 0.6141 0.6517 0.6879 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.8849 0.9032 0.9192 0.9192 0.9332
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6	0.5398 0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.8849 0.9032 0.9032 0.9192 0.9332	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.9049 0.9049 0.9207 0.9345 0.9463	0.5478 0.6255 0.6628 0.7324 0.742 0.7642 0.7939 0.8212 0.8461 0.8686 0.8888 0.9066 0.9222 0.9357 0.9474	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485 0.8708 0.8907 0.9082 0.9236 0.9236 0.9370 0.9484	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251 0.9382 0.9382 0.9382	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394 0.9505	0.5636 0.6026 0.6772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406 0.9515	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418 0.9525	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8897 0.9162 0.9306 0.9429 0.9535	0.5753 0.6141 0.6517 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441 0.9545	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849 0.9032 0.9192 0.9132 0.9332 0.9452 0.9554
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7	0.5398 0.6793 0.6554 0.6915 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.8849 0.9032 0.9192 0.9332 0.9332 0.9452 0.9554	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.9049 0.9207 0.9345 0.9463 0.9463	0.5478 0.6255 0.6628 0.7324 0.742 0.7642 0.7939 0.8212 0.8461 0.8686 0.9066 0.9222 0.9357 0.9474 0.9573	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485 0.8708 0.8708 0.8907 0.9082 0.9236 0.9236 0.9370 0.9484 0.9582	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251 0.9382 0.9382 0.9382 0.9382 0.9495	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394 0.9505 0.9599	0.5636 0.6026 0.6772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406 0.9515 0.9608	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418 0.9525 0.9616	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8897 0.9162 0.9306 0.9429 0.9535 0.9625	0.5753 0.6141 0.6517 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441 0.9545 0.9633	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849 0.9032 0.9192 0.9132 0.9332 0.9452 0.9554 0.9641
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8	0.5398 0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849 0.9032 0.9192 0.9332 0.9452 0.9554 0.9641	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.9049 0.9207 0.9345 0.9463 0.9564	0.5478 0.6255 0.6628 0.7324 0.742 0.7642 0.7939 0.8212 0.8461 0.8686 0.9888 0.9066 0.9222 0.9357 0.9474 0.9573	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485 0.8708 0.8907 0.9082 0.9236 0.9236 0.9370 0.9484 0.9582 0.9664	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251 0.9382 0.9495 0.9495 0.9591 0.9591	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394 0.9505 0.9599 0.9599	0.5636 0.6026 0.6772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406 0.9515 0.9608	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418 0.9525 0.9616 0.9693	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997 0.9162 0.9306 0.9429 0.9535 0.9625 0.9699	0.5753 0.6141 0.6517 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441 0.9545 0.9633 0.9706	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849 0.9032 0.9192 0.9332 0.9192 0.9332 0.9452 0.9554 0.9641 0.9713
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9	0.5398 0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.9032 0.9192 0.9332 0.9322 0.9452 0.9554 0.9554 0.9641	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.9049 0.9207 0.9345 0.9463 0.9564 0.9564 0.9649 0.9719	0.5478 0.5871 0.6255 0.6628 0.7324 0.7642 0.7939 0.8212 0.8461 0.8686 0.9888 0.9066 0.9222 0.9357 0.9474 0.9573 0.9656 0.9726	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485 0.8485 0.8708 0.8907 0.9082 0.9236 0.9370 0.9484 0.9582 0.9582 0.9664 0.9732	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251 0.9382 0.9495 0.9495 0.9591 0.9591 0.9671	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394 0.9505 0.9599 0.9599 0.9678 0.9744	0.5636 0.6026 0.6772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406 0.9515 0.9608 0.9686 0.9750	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418 0.9525 0.9616 0.9693 0.9756	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997 0.9162 0.9306 0.9429 0.9429 0.9535 0.9625 0.9699 0.9761	0.5753 0.6141 0.6517 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441 0.9545 0.9633 0.9706 0.9767	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849 0.9032 0.9192 0.9192 0.9332 0.9452 0.9554 0.9554 0.9641 0.9713
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2	0.5398 0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.9032 0.9192 0.9322 0.9322 0.9452 0.9554 0.9554 0.9641 0.9713 0.9772	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.710 0.8186 0.8438 0.8665 0.8869 0.9049 0.9207 0.9345 0.9345 0.9345 0.9463 0.9564 0.9564 0.9649 0.9719	0.5478 0.5871 0.6255 0.6628 0.7324 0.7642 0.7939 0.8212 0.8461 0.8686 0.8888 0.9066 0.9222 0.9357 0.9474 0.9573 0.9656 0.9726	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7673 0.8238 0.8485 0.8708 0.8907 0.9082 0.9236 0.9236 0.9370 0.9484 0.9582 0.9664 0.9732 0.9788	0.5557 0.5948 0.6331 0.700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251 0.9382 0.9495 0.9591 0.9671 0.9738 0.9793	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394 0.9505 0.9599 0.9678 0.9744 0.9798	0.5636 0.6026 0.6406 0.772 0.7123 0.7454 0.8051 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406 0.9515 0.9608 0.9686 0.9750 0.9803	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418 0.9525 0.9616 0.9693 0.9756 0.9808	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997 0.9162 0.9306 0.9429 0.9535 0.9625 0.9629 0.9699 0.9761 0.9812	0.5753 0.6141 0.6517 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441 0.9545 0.9633 0.9706 0.9767 0.9817	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849 0.9032 0.9192 0.9322 0.9322 0.9452 0.9554 0.9554 0.9554 0.9713 0.9772 0.9821
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1	0.5398 0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.8849 0.9032 0.9192 0.9322 0.9192 0.9332 0.9452 0.9554 0.9641 0.9641 0.9713 0.9772 0.9821	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.9049 0.9207 0.9345 0.9463 0.9463 0.9564 0.9564 0.9649 0.9719 0.9778 0.9778	0.5478 0.5871 0.6255 0.6628 0.7324 0.7642 0.7939 0.8212 0.8461 0.8686 0.8888 0.9066 0.9222 0.9357 0.9474 0.9573 0.9656 0.9726 0.9783	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485 0.8708 0.8485 0.8708 0.8907 0.9082 0.9236 0.9370 0.9484 0.9582 0.9664 0.9732 0.9788 0.9788 0.9834	0.5557 0.5948 0.6331 0.7054 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251 0.9382 0.9495 0.9495 0.9591 0.9671 0.9738 0.9793 0.9793	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394 0.9505 0.9394 0.9505 0.9599 0.9678 0.9744 0.9798	0.5636 0.6026 0.6772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406 0.9515 0.9608 0.9686 0.9750 0.9803 0.9846	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418 0.9525 0.9616 0.9693 0.9756 0.9808 0.9850	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997 0.9162 0.9306 0.9429 0.9355 0.9625 0.9625 0.9699 0.9761 0.9812 0.9854	0.5753 0.6141 0.6517 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441 0.9545 0.9633 0.9706 0.9767 0.9817 0.9857	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.8849 0.9032 0.9192 0.9192 0.9332 0.9452 0.9554 0.9554 0.9641 0.9713 0.9772 0.9821 0.9861
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2	0.5398 0.6179 0.6554 0.6915 0.7257 0.7580 0.7881 0.8459 0.8413 0.8643 0.8849 0.9032 0.9032 0.9192 0.9322 0.9354 0.9554 0.9554 0.9554 0.9641 0.9713 0.9772 0.9821	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.9049 0.9207 0.9345 0.9463 0.9463 0.9564 0.9564 0.9564 0.9719 0.9778 0.9778 0.9826	0.5478 0.6255 0.6628 0.7324 0.7642 0.7642 0.7939 0.8212 0.8461 0.8686 0.8888 0.9066 0.9222 0.9357 0.9474 0.9573 0.9474 0.9573 0.9656 0.9726 0.9783 0.9830	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485 0.8708 0.8907 0.9082 0.9082 0.9236 0.9370 0.9484 0.9582 0.9664 0.9732 0.9788 0.9834 0.9831	0.5557 0.5948 0.6331 0.7054 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8251 0.8925 0.9099 0.9251 0.9382 0.9495 0.9495 0.9591 0.9671 0.9671 0.9738 0.9793 0.9793 0.9838 0.9875	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394 0.9505 0.9394 0.9505 0.9599 0.9678 0.9744 0.9798 0.9782	0.5636 0.6026 0.6772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406 0.9515 0.9608 0.9686 0.9750 0.9803 0.9803 0.9846 0.9881	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418 0.9525 0.9616 0.9693 0.9756 0.9808 0.9850 0.9884	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997 0.9162 0.9306 0.9429 0.9535 0.9625 0.9625 0.9699 0.9761 0.9812 0.9854 0.9887	0.5753 0.6141 0.6517 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441 0.9545 0.9633 0.9706 0.9767 0.9817 0.9857 0.9890	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.9032 0.9192 0.9192 0.9332 0.9452 0.9452 0.9554 0.9641 0.9713 0.9772 0.9821 0.9861 0.9893
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3	0.5398 0.6793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849 0.9032 0.9032 0.9192 0.9322 0.9354 0.9554 0.9554 0.9641 0.9713 0.9772 0.9821 0.9861 0.9893	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.9049 0.9049 0.9207 0.9345 0.9463 0.9564 0.9564 0.9564 0.9719 0.9778 0.9778 0.9826 0.9864 0.9864	0.5478 0.6255 0.6628 0.7324 0.742 0.7642 0.7939 0.8212 0.8461 0.8686 0.8888 0.9066 0.9222 0.9357 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485 0.8708 0.8907 0.9082 0.9082 0.9236 0.9370 0.9484 0.9582 0.9582 0.9664 0.9732 0.9664 0.9732 0.9788 0.9834 0.9834 0.9871 0.9901	0.5557 0.5948 0.6331 0.7054 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251 0.9382 0.9495 0.9495 0.9591 0.9671 0.9738 0.9793 0.9838 0.9793 0.9838 0.9875 0.9904	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394 0.9505 0.9394 0.9505 0.9599 0.9678 0.9744 0.9798 0.9842 0.9878 0.9878	0.5636 0.6026 0.6772 0.7123 0.7454 0.7764 0.8051 0.8315 0.8554 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406 0.9515 0.9608 0.9608 0.9686 0.9750 0.9803 0.9846 0.9881 0.9881	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418 0.9525 0.9616 0.9693 0.9756 0.9808 0.9850 0.9884 0.9851	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997 0.9162 0.9306 0.9429 0.9305 0.9429 0.9535 0.9625 0.9625 0.9699 0.9761 0.9812 0.9854 0.9887 0.9913	0.5753 0.6141 0.6517 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441 0.9545 0.9633 0.9706 0.9767 0.9817 0.9857 0.9890 0.9916	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.9032 0.9192 0.9322 0.9332 0.9452 0.9554 0.9641 0.9713 0.9772 0.9821 0.9821 0.9881 0.9893
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4	0.5398 0.6793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849 0.9032 0.9032 0.9192 0.9332 0.9192 0.9332 0.9452 0.9554 0.9554 0.9641 0.9713 0.9772 0.9821 0.9861 0.9893 0.9918	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.9049 0.9049 0.9207 0.9345 0.9463 0.9463 0.9564 0.9564 0.9564 0.9719 0.9778 0.9778 0.9826 0.9864 0.9896 0.9896	0.5478 0.6255 0.6628 0.7324 0.7642 0.7642 0.7939 0.8212 0.8461 0.8686 0.9066 0.9222 0.9357 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485 0.8708 0.8907 0.9082 0.9236 0.9370 0.9484 0.9582 0.9664 0.9732 0.9664 0.9732 0.9664 0.9732 0.9788 0.9834 0.9834 0.9831 0.9871 0.9901	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251 0.9382 0.9495 0.9382 0.9495 0.9591 0.9671 0.9738 0.9738 0.9738 0.9838 0.9875 0.9904 0.9927	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394 0.9505 0.9394 0.9505 0.9599 0.9678 0.9744 0.9798 0.9744 0.9798 0.9842 0.9878 0.9906 0.9929	0.5636 0.6026 0.6406 0.772 0.7123 0.7454 0.8051 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406 0.9515 0.9608 0.9515 0.9608 0.9686 0.9750 0.9803 0.9846 0.9881 0.9881 0.9909 0.9931	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418 0.9525 0.9616 0.9693 0.9756 0.9808 0.9850 0.9884 0.9850 0.9884 0.9911 0.9932	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997 0.9162 0.9306 0.9429 0.9305 0.9625 0.9625 0.9625 0.9699 0.9761 0.9812 0.9854 0.9887 0.9887 0.9913	0.5753 0.6141 0.6517 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441 0.9545 0.9633 0.9706 0.9767 0.9817 0.9857 0.9857 0.9890 0.9916 0.9936	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.9032 0.9192 0.9322 0.9322 0.9452 0.9554 0.9641 0.9713 0.9772 0.9821 0.9861 0.9881 0.9883 0.9918
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5	0.5398 0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.8849 0.9032 0.9192 0.9332 0.9192 0.9332 0.9452 0.9554 0.9554 0.9554 0.9641 0.9713 0.9772 0.9821 0.9821 0.9881 0.9893 0.9918 0.9938	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.9049 0.9049 0.9207 0.9345 0.9463 0.9564 0.9564 0.9564 0.9564 0.9719 0.9778 0.9778 0.9826 0.9826 0.9886 0.9896 0.9920 0.9920	0.5478 0.6255 0.6628 0.7324 0.7642 0.7642 0.7939 0.8212 0.8461 0.8686 0.9066 0.9222 0.9357 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485 0.8708 0.8907 0.9082 0.9236 0.9370 0.9484 0.9582 0.9664 0.9732 0.9664 0.9732 0.9664 0.9732 0.9788 0.9834 0.9834 0.9871 0.9901 0.9925 0.9943	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251 0.9382 0.9495 0.9495 0.9591 0.9671 0.9738 0.9738 0.9738 0.9793 0.9838 0.9875 0.9904	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394 0.9505 0.9599 0.9678 0.9599 0.9678 0.9744 0.9798 0.9744 0.9798 0.9878 0.9878 0.9906 0.9929 0.9946	0.5636 0.6026 0.6406 0.772 0.7123 0.7454 0.8051 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406 0.9515 0.9608 0.9515 0.9608 0.9550 0.9686 0.9750 0.9803 0.9846 0.9881 0.9909 0.9931 0.9948	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418 0.9525 0.9616 0.9693 0.9756 0.9808 0.9850 0.9884 0.9850 0.9884 0.9911 0.9932 0.9949	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997 0.9162 0.9306 0.9429 0.9335 0.9625 0.9625 0.9699 0.9761 0.9812 0.9854 0.9854 0.9887 0.9913 0.9934 0.9934	0.5753 0.6141 0.6517 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441 0.9545 0.9633 0.9706 0.9767 0.9817 0.9857 0.9857 0.9890 0.9916 0.9936	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.9032 0.9192 0.9132 0.9322 0.9452 0.9554 0.9641 0.9713 0.9772 0.9821 0.9821 0.9881 0.9893 0.9918 0.9938
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4	0.5398 0.6793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8849 0.9032 0.9032 0.9192 0.9332 0.9192 0.9332 0.9452 0.9554 0.9554 0.9641 0.9713 0.9772 0.9821 0.9861 0.9893 0.9918	0.5438 0.5832 0.6217 0.6591 0.7291 0.7611 0.7910 0.8186 0.8438 0.8665 0.8869 0.9049 0.9049 0.9207 0.9345 0.9463 0.9463 0.9564 0.9564 0.9564 0.9719 0.9778 0.9778 0.9826 0.9864 0.9896 0.9896	0.5478 0.6255 0.6628 0.7324 0.7642 0.7642 0.7939 0.8212 0.8461 0.8686 0.9066 0.9222 0.9357 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474 0.9573 0.9474	0.5517 0.5910 0.6293 0.6664 0.7019 0.7357 0.7673 0.7967 0.8238 0.8485 0.8708 0.8907 0.9082 0.9236 0.9370 0.9484 0.9582 0.9664 0.9732 0.9664 0.9732 0.9664 0.9732 0.9788 0.9834 0.9834 0.9831 0.9871 0.9901	0.5557 0.5948 0.6331 0.6700 0.7054 0.7389 0.7704 0.7995 0.8264 0.8508 0.8729 0.8925 0.9099 0.9251 0.9382 0.9495 0.9382 0.9495 0.9591 0.9671 0.9738 0.9738 0.9738 0.9838 0.9875 0.9904 0.9927	0.5596 0.5987 0.6368 0.736 0.7088 0.7422 0.7734 0.8023 0.8289 0.8531 0.8749 0.8944 0.9115 0.9265 0.9394 0.9505 0.9394 0.9505 0.9599 0.9678 0.9744 0.9798 0.9744 0.9798 0.9842 0.9878 0.9906	0.5636 0.6026 0.6406 0.772 0.7123 0.7454 0.8051 0.8315 0.8554 0.8770 0.8962 0.9131 0.9279 0.9406 0.9515 0.9608 0.9515 0.9608 0.9686 0.9750 0.9803 0.9846 0.9881 0.9881 0.9909 0.9931	0.5675 0.6064 0.6443 0.7157 0.7486 0.7794 0.8078 0.8340 0.8577 0.8790 0.8980 0.9147 0.9292 0.9418 0.9525 0.9616 0.9693 0.9756 0.9808 0.9850 0.9884 0.9850 0.9884 0.9911 0.9932	0.5714 0.6103 0.6480 0.7190 0.7517 0.7823 0.8106 0.8365 0.8599 0.8810 0.8997 0.9162 0.9306 0.9429 0.9305 0.9625 0.9625 0.9625 0.9699 0.9761 0.9812 0.9854 0.9887 0.9887 0.9913	0.5753 0.6141 0.6517 0.7224 0.7549 0.7852 0.8133 0.8389 0.8621 0.8830 0.9015 0.9177 0.9319 0.9441 0.9545 0.9633 0.9706 0.9767 0.9817 0.9857 0.9857 0.9890 0.9916 0.9936	0.5793 0.6179 0.6554 0.7257 0.7580 0.7881 0.8159 0.8413 0.8643 0.8643 0.9032 0.9192 0.9322 0.9322 0.9452 0.9554 0.9641 0.9713 0.9772 0.9821 0.9861 0.9881 0.9883 0.9918

2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0.9987
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	0.9990

The values for both z-tables correspond with the same information that is illustrated in a z-distribution graph but actually say more about the specific likelihood of an event occurring.

If you are provided with a negative z-score, you use the negative z-table to find the area under the curve to the left of the given z-score. This area under the curve is equal to the proportion of all z-scores in this range. When you have a positive z-score, the positive z-table lets you find the area under the curve to the left of that given z-score. This area under the curve is equal to the proportion of all z-scores in this range.

TERM TO KNOW

Z-Table

Table for looking up the area starting at z = 0 for a positive z-score.

3. Using Z-Tables to Find Probability

Take a look at ACT scores to illustrate this. The American College Test, or ACT, is taken by high school students all across the United States as a means of determining their aptitude for attending college. Let's suppose in this instance that the mean score for the population is 21, and the standard deviation in this case is 5. How will you determine the probability that a score would fall within a particular range?

3a. Higher

Consider the probability that the score would be higher than 30. How do you determine what that value is? The first thing you do is use the z-score formula to figure out what the z-score is. In this case, it is the difference between 30 and 21, which is 9, divided by the standard deviation of 5, which gives you a z-score of 1.8. If you look at the z-table below, that gives you a probability value of 0.9641.

Probability of an ACT score > 30 Mean = 21 Standard Deviation = 5

$$z = \frac{30 - 21}{5} = 1.80$$

z-table for positive z-values

		ve z-values	·								
z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.5398
0.	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.5793
0.	2 0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	0.6179
0.	3 0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	0.6554
0.	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	0.6915
0.	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	0.7257
0.	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	0.7580
0.	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	0.7881
0.	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	0.8159
0.	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	0.8413
:	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	0.8643
1.3	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	0.8849
1.	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	0.9032
1.3	3 0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	0.9192
1.	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	0.9332
1.	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	0.9452
1.	6 0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	0.9554
1.	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	0.9641
1.	8 0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	0.9713
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	0.9772
	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0.9821

Now, what does that tell you? It tells you that the area to the left of that point is 0.9641, or roughly 96% of the area under the curve falls to the left of z equal to 1.8. To determine the probability that the score is greater than 30, you're interested in the difference between 0.9641 and 1, which gives you a probability of 0.0359.

Probability = $(1 - 0.9641) = 0.359 \text{ or } \sim 3.6\%$

3b. Range

What if you're interested in the probability that a score falls between 23 and 27? In this instance, you need to calculate two different z-scores, one for 23 and one for 27. Locate both of these z-scores on the z-table. Then subtract the difference between the greater value and the lower value. That is the probability that a randomly drawn score would fall between the range of 23 and 27.

Probability of a score between 23 and 27

$$z = \frac{27 - 21}{5} = 1.20$$
$$z = \frac{23 - 21}{5} = 0.40$$

Probability = 0.8849 - 0.6554 = 0.2295 or ~23%

z-table for positive z-values

0.01	0.02	0.00							
	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
00 0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.5398
98 0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.5793
93 0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	0.6179
79 0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	0.6554
54 0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	0.6915
15 0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	0.7257
57 0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	0.7580
80 0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	0.7881
81 0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	0.8159
<mark>59</mark> 0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	0.8413
13 0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	0.8643
43 0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	0.8849
49 0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	0.9032
32 0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	0.9192
92 0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	0.9332
<mark>32</mark> 0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	0.9452
	398 0.5438 793 0.5832 179 0.6217 554 0.6591 915 0.6950 257 0.7291 580 0.7611 381 0.7910 159 0.8186 413 0.8438 543 0.8665 392 0.9045 192 0.9207	398 0.5438 0.5478 793 0.5832 0.5871 179 0.6217 0.6255 54 0.6591 0.6628 915 0.6950 0.6985 257 0.7291 0.7324 580 0.7611 0.7642 381 0.7910 0.7939 159 0.8186 0.8212 413 0.8438 0.8461 543 0.8665 0.8686 392 0.9049 0.9066 192 0.9207 0.9222	398 0.5438 0.5478 0.5517 793 0.5832 0.5871 0.5910 179 0.6217 0.6255 0.6293 54 0.6591 0.6628 0.6664 15 0.6950 0.6985 0.7019 257 0.7291 0.7324 0.7357 680 0.7611 0.7642 0.7673 811 0.7910 0.7939 0.7967 159 0.8186 0.8212 0.8238 413 0.8438 0.8461 0.8485 643 0.8665 0.8686 0.8708 643 0.8469 0.8888 0.8907 032 0.9049 0.9066 0.9082 042 0.9207 0.9222 0.9236	398 0.5438 0.5478 0.5517 0.5557 793 0.5832 0.5871 0.5910 0.5948 179 0.6217 0.6255 0.6293 0.6331 54 0.6591 0.6628 0.6664 0.6700 55 0.7291 0.7324 0.7357 0.7389 580 0.7611 0.7642 0.7673 0.7704 581 0.7910 0.7939 0.7967 0.7995 582 0.8186 0.8212 0.8238 0.8264 413 0.8438 0.8461 0.8485 0.8508 543 0.8665 0.8686 0.8708 0.8729 544 0.9049 0.9066 0.9082 0.9099 543 0.9207 0.9222 0.9236 0.9251	398 0.5438 0.5478 0.5517 0.5557 0.5596 793 0.5832 0.5871 0.5910 0.5948 0.5987 179 0.6217 0.6255 0.6293 0.6331 0.6368 54 0.6591 0.6628 0.6664 0.6700 0.6736 515 0.6950 0.6985 0.7019 0.7054 0.7088 257 0.7291 0.7324 0.7357 0.7389 0.7422 680 0.7611 0.7642 0.7673 0.7704 0.7734 811 0.7910 0.7939 0.7967 0.7995 0.8023 159 0.8186 0.8212 0.8238 0.8264 0.8289 413 0.8438 0.8461 0.8485 0.8508 0.8531 643 0.8665 0.8686 0.8708 0.8729 0.8749 643 0.9049 0.9066 0.9082 0.9099 0.9115 643 0.9207 0.9222 0.9236 0.9251	398 0.5438 0.5478 0.5517 0.5557 0.5596 0.5636 793 0.5832 0.5871 0.5910 0.5948 0.5987 0.6026 793 0.6217 0.6255 0.6293 0.6331 0.6368 0.6406 794 0.6591 0.6628 0.6664 0.6700 0.6736 0.6772 915 0.6950 0.6985 0.7019 0.7054 0.7088 0.7123 927 0.7291 0.7324 0.7357 0.7389 0.7422 0.7454 938 0.7611 0.7642 0.7673 0.7704 0.7734 0.7764 941 0.7910 0.7939 0.7967 0.7995 0.8023 0.8051 943 0.8438 0.8461 0.8485 0.8508 0.8531 0.8554 943 0.8665 0.8686 0.8708 0.8729 0.8749 0.8770 943 0.8665 0.8888 0.8907 0.8925 0.8944 0.8962 <th< th=""><th>3980.54380.54780.55170.55570.55960.56360.56757930.58320.58710.59100.59480.59870.60260.60647930.62170.62550.62930.63310.63680.64060.64437940.65910.66280.66640.67000.67360.67720.68089150.69500.69850.70190.70540.70880.71230.71579270.72910.73240.73570.73890.74220.74540.74869380.76110.76420.76730.77040.77340.77640.77949410.81860.82120.82380.82640.82890.83150.83409430.84380.84610.84850.85080.85310.85540.85779430.86650.86860.87080.87290.87490.87700.87909440.89650.88880.89070.89250.89440.89620.89809450.92070.92220.92360.92510.92650.92790.9292</th><th>3980.54380.54780.55170.55570.55960.56360.56750.57147930.58320.58710.59100.59480.59870.60260.60640.61037930.62170.62550.62930.63310.63680.64060.64430.64806540.65910.66280.66640.67000.67360.67720.68080.68446150.69500.69850.70190.70540.70880.71230.71570.71906270.72910.73240.73570.73890.74220.74540.74860.75176300.76110.76420.76730.77040.77340.77640.77940.78236310.79100.79390.79670.79950.80230.80510.80780.816664330.84610.84850.85080.85310.85540.85770.859964330.86650.86860.87080.87290.87490.87700.87900.881064340.86650.88880.89070.89250.89440.89620.89800.89976320.90490.90660.90820.90990.91150.91310.91470.91626320.92070.92220.92360.92510.92650.92790.92920.9306</th><th>3980.54380.54780.55170.55570.55960.56360.56750.57140.57537930.58320.58710.59100.59480.59870.60260.60640.61030.61411790.62170.62550.62930.63310.63680.64060.64430.64800.65175540.65910.66280.66640.67000.67360.67720.68080.68440.68799150.69500.69850.70190.70540.70880.71230.71570.71900.72246300.76110.76420.76730.77040.77340.77640.77940.78230.78526310.79100.79390.79670.79950.80230.80510.80780.81060.81336430.84610.84850.85080.85310.85540.87770.85990.86216430.86650.86860.87080.87290.87700.87900.88100.88306430.86650.86860.87080.87290.87490.87700.87900.88100.88306430.86650.88880.89070.89250.89440.89620.89800.89970.90156320.90490.90660.90820.90990.91150.91310.91470.91620.91716320.92070.92220.92360.92510.92650.92790.92920.93060.9319</th></th<>	3980.54380.54780.55170.55570.55960.56360.56757930.58320.58710.59100.59480.59870.60260.60647930.62170.62550.62930.63310.63680.64060.64437940.65910.66280.66640.67000.67360.67720.68089150.69500.69850.70190.70540.70880.71230.71579270.72910.73240.73570.73890.74220.74540.74869380.76110.76420.76730.77040.77340.77640.77949410.81860.82120.82380.82640.82890.83150.83409430.84380.84610.84850.85080.85310.85540.85779430.86650.86860.87080.87290.87490.87700.87909440.89650.88880.89070.89250.89440.89620.89809450.92070.92220.92360.92510.92650.92790.9292	3980.54380.54780.55170.55570.55960.56360.56750.57147930.58320.58710.59100.59480.59870.60260.60640.61037930.62170.62550.62930.63310.63680.64060.64430.64806540.65910.66280.66640.67000.67360.67720.68080.68446150.69500.69850.70190.70540.70880.71230.71570.71906270.72910.73240.73570.73890.74220.74540.74860.75176300.76110.76420.76730.77040.77340.77640.77940.78236310.79100.79390.79670.79950.80230.80510.80780.816664330.84610.84850.85080.85310.85540.85770.859964330.86650.86860.87080.87290.87490.87700.87900.881064340.86650.88880.89070.89250.89440.89620.89800.89976320.90490.90660.90820.90990.91150.91310.91470.91626320.92070.92220.92360.92510.92650.92790.92920.9306	3980.54380.54780.55170.55570.55960.56360.56750.57140.57537930.58320.58710.59100.59480.59870.60260.60640.61030.61411790.62170.62550.62930.63310.63680.64060.64430.64800.65175540.65910.66280.66640.67000.67360.67720.68080.68440.68799150.69500.69850.70190.70540.70880.71230.71570.71900.72246300.76110.76420.76730.77040.77340.77640.77940.78230.78526310.79100.79390.79670.79950.80230.80510.80780.81060.81336430.84610.84850.85080.85310.85540.87770.85990.86216430.86650.86860.87080.87290.87700.87900.88100.88306430.86650.86860.87080.87290.87490.87700.87900.88100.88306430.86650.88880.89070.89250.89440.89620.89800.89970.90156320.90490.90660.90820.90990.91150.91310.91470.91620.91716320.92070.92220.92360.92510.92650.92790.92920.93060.9319

What if you're looking at some areas to the left of the mean? Suppose you're interested in the probability that a score falls between 15 and 20. You will run the same type of calculations. The z-score for a 20 would be equal to a negative 0.2. The z-score for 15 would be equal to negative 1.2. Once again, you figure out the probabilities associated with each value by finding the difference. This works out to be a probability of 0.3056.

Probability of a score between 15 and 20

$$z = \frac{20 - 21}{5} = -0.20$$
$$z = \frac{15 - 21}{5} = -1.20$$

Probability = 0.4207-0.1151 = 0.3056 or ~31%

See the z-table for this scenario.

+

z-table for negative z-values

2-table		ive z-value									
z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
-3	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	0.0013
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036	0.0035
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048	0.0047
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064	0.0062
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084	0.0082
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110	0.0107
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143	0.0139
-2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183	0.0179
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233	0.0228
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294	0.0287
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367	0.0359
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455	0.0446
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559	0.0548
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681	0.0668
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823	0.0808
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985	0.0968
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170	0.1151
-1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379	0.1357
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611	0.1587
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867	0.1841
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148	0.2119
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451	0.2420
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776	0.2743
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121	0.3085
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483	0.3446
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859	0.3821
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247	0.4207
0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641	0.4602

3c. Lower

Lastly, look at the probability that a score will be less than 20. Here you simply need to find the z-score for 20, which is equal to -0.20. The probability for this value is 0.4207, which tells you that's the probability that a score would be below 20, based upon a normal distribution.

Probability of a score less than 20

$$z = \frac{20 - 21}{5} = -0.20$$

Probability = 0.4207 or ~42%

See the z-table for this scenario.

2-table		ive z-value									
z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
-3	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	0.0013
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036	0.0035
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048	0.0047
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064	0.0062
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084	0.0082
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110	0.0107
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143	0.0139
-2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183	0.0179
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233	0.0228
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294	0.0287
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367	0.0359
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455	0.0446
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559	0.0548
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681	0.0668
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823	0.0808
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985	0.0968
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170	0.1151
-1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379	0.1357
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611	0.1587
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867	0.1841
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148	0.2119
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451	0.2420
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776	0.2743
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121	0.3085
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483	0.3446
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859	0.3821
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247	0.4207
0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641	0.4602

4. Using Z-Tables to Find Values

What if you're interested in determining the value of a variable based upon a predetermined probability? Hypothetically speaking, let's look at the miles driven per year by the American driver. Say that in the population that you're looking at, the mean miles driven per year is 16,550, with a population standard deviation of 2,100.

Suppose you were interested in finding the number of miles driven per year in which there were only 1.5% of all observations below that value. How do you determine that figure? Well, you establish your probability of 1.5%. Then, divide that by 100 to arrive at a probability equal to 0.015, which is the value you will look up in your z-table.

See the z-table for this scenario.

z-table for negative z-values

z-table													
z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10		
-3	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010	0.0010		
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	0.0013		
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019	0.0019		
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	0.0026		
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036	0.0035		
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048	0.0047		
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064	0.0062		
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084	0.0082		
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110	0.0107		
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143	0.0139		
-2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183	0.0179		
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233	0.0228		
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294	0.0287		
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367	0.0359		
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455	0.0446		
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559	0.0548		
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681	0.0668		
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823	0.0808		
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985	0.0968		
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170	0.1151		
-1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379	0.1357		

You scan through the numbers until you find the value that's equal to 0.015, which is going to be in the negative side of the z-table. Look at the columns and the rows, and you will find the value of 0.015 happens to have a z-value of -2.17. Now, plug that into the z-score formula and solve for the value:

 $\frac{?-16,550}{2,100} = -2.17$?-16,550 = -4,557 ?=11,993

The result is 11,993 miles per year. Only 1.5% of drivers drive fewer than that number of miles per year.

Suppose that you have a probability of 69.5%. Once again, divide this value by 100 to arrive at a probability of 0.695, and scan the z-table to find that 0.695 value. It turns out that the z-value is equal to 0.51.

See the z-table for this scenario.

z-table	e for positi	ve z-values	6								
z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.5398
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.5793
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	0.6179
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	0.6554
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	0.6915
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	0.7257
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	0.7580
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	0.7881
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	0.8159
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	0.8413
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	0.8643

To solve for your value, you look at whatever the value is minus 16,550. We take the difference and divide it by 2,100 to get 0.51. So, if you do a little cross multiplication, you arrive at a value of 17,621 miles per year, in which case 69.5% of drivers would drive less than that.

 $\frac{?-16,550}{2,100} = 0.51$?-16,550 = 1,071 ? = 17,621

SUMMARY

In this lesson, you reviewed how **normal distributions and z-scores** relate, which led into **z-tables**. These are a way to look up more detailed information that a graph can give. You also saw **how z-tables are used to find probability** in several examples involving determining the likelihood of a range of values of a normally distributed variable occurring—whether a score would be **higher** or **lower** than a particular value or fall within a particular **range** of values. You can also reverse it and look at the probability first, **using z-tables to find values** of variables.

Source: THIS TUTORIAL WAS AUTHORED BY DAN LAUB FOR SOPHIA LEARNING. PLEASE SEE OUR TERMS OF USE.

TERMS TO KNOW

Z-Table

Table for looking up the area starting at z = 0 for a positive z-score.