Sophia

Work, Rate, and Time in a System of Equations

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to solve a work, rate, and time, problem by using a system of equations. Specifically, this lesson will cover:

1. Work, Rate, and Time

The relationship between work, rate, and time can be modeled using a similar equation to distance, rate, and time:

FORMULA TO KNOW

Work, Rate, and Time

$work = rate \cdot time$

We multiply the rate at which someone completes work, or a particular job, by the amount of time they spent working.

2. Combined Rates

If two people are working together, we can add their rates together and reflect this in the formula for work, rate, and time.

↪ EXAMPLE If a professor can grade 10 papers in one hour, and her teaching assistant can grade 7 papers in one hour, we know that their combined rate is 17 papers per hour.

In general, if two people are working together, we use (r_1+r_2) as the rate in the formula, to show that Rate 1 and Rate 2 are combined.

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3. Solving for Combined Rate Using System of Equations

Again, to solve a work, rate, time problem in the real world, we'll need to define our variables and create a system of equations.

Let's return to the professor and teaching assistant scenario.

↪ EXAMPLE A professor can grade 80 papers in the same amount of time it takes for her teaching assistant to grade 60 papers. The teaching assistant grades 1 less paper per hour than the professor. How many papers can the two grade in one hour?

Let's use what we know about work, rate, and time (along with combined rate) to create some equations based on what we know. First, let's define our variables. Let's say r_1 is the hourly rate of the professor, and r_2 is the hourly rate of the teaching assistant.

$$r_1 = \text{professor's rate}$$
$$r_2 = \text{teaching assistant's rate}$$

We can use the given information to find the following work, rate, and time equations. Note we are able to create a third equation by adding the first two equations together to find the combined rate.

$$80 = r_1 \cdot t$$
$$60 = r_2 \cdot t$$
$$140 = (r_1 + r_2) \cdot t$$

We also know that the teaching assistant grades one less paper per hour, so we can subtract 1 from the professor's rate to represent the teaching assistant's rate relative to the professor. This is shown below:

$$r_2 = r_1 - 1$$

We can use this equivalent expression for r_2 in one of the other equations in our system. Instead of writing r_2, we will write $r_1 - 1$. This will allow us to make further substitutions and eventually solve for an unknown variable. Use the equation $60 = r_2 \cdot t$ since the only variable is r_2.

$60 = r_2 \cdot t$	Using this equation, substitute r_2 for $r_1 - 1$

$60 = (r_1 - 1) \cdot t$	Distribute *t*
$60 = r_1 t - t$	From our first equation, substitute 80 for $r_1 t$
$60 = 80 - t$	Subtract 80 from both sides
$-20 = -t$	Divide both sides by -1
$20 = t$	Solve for *t*

Now that we have a value for *t*, we can use this in another equation in our system to solve for other variables. We eventually want to know how many papers the professor and assistant can grade in one hour. We already have an equation for this in our system; it is the equation we found by adding two equations together to show the combined rate:

$$140 = (r_1 + r_2) \cdot t$$

We can simply plug in 20 for *t*, and solve for $(r_1 + r_2)$. Notice that we don't necessarily need to solve for the two rates individually because we are interested in what their combined rate is:

$140 = (r_1 + r_2) \cdot t$	Using the combined rate equation, substitute 20 in for *t*
$140 = (r_1 + r_2) \cdot 20$	Divide both sides by 20
$7 = (r_1 + r_2)$	Our Solution

Together, the professor and teaching assistant can grade 7 papers in one hour.

4. Solving for Individual Rates Using System of Equations

Now we'll look at an example where we find individual rates per hour.

EXAMPLE Suppose Kate and Jenny work in a bicycle shop. Jenny can repair 48 bikes in the same amount of time that it takes Kate to repair 32 bikes. Also, Jenny can repair 2 more bikes per hour than Kate can. We want to know the rate per hour that Jenny repairs bikes and the rate per hour that Kate repairs bikes.

Let's start by defining our two variables for what we want to know. Let's define r_1 as Jenny's rate for repairing bikes and r_2 as Kate's rate for repairing bikes.

$r_1 = \text{Jenny's rate}$
$r_2 = \text{Kate's rate}$

The first thing that we know is that the amount of time that it takes Jenny to repair 48 bikes is the same as the amount of time that it takes Kate to repair 32 bikes. Thinking about our relationship between work, rate, and time, we know that we can represent time as being equal to the work divided by the rate. We can define the amount of time that it takes Jenny to repair bikes as the work over her rate. She can repair 48 bikes and that will be over her rate, which is r_1.

Jenny's time: $t = \frac{48}{r_1}$

We can define the amount of time that it takes Kate to repair bikes as the work over her rate. She can repair 32 bikes and that will be over her rate which is r_2.

Kate's time: $t = \frac{32}{r_2}$

We know that it takes them the same amount of time to do this work, so we can set the equations equal to each other: This will be the first equation in our system of equations.

$$\frac{48}{r_1} = \frac{32}{r_2}$$

The second thing we know is that Jenny can repair 2 more bikes per hour than Kate can. This means that Jenny's rate is going to be equal to whatever Kate's rate is plus 2. This will be the second equation in our system of equations.

$$r_1 = r_2 + 2$$

Now we have two equations for our system of equations:

$$\frac{48}{r_1} = \frac{32}{r_2}$$
$$r_1 = r_2 + 2$$

Because we have a variable in one of the equations already isolated, the substitution method is going to be the easiest method to use to solve this system. Let's go ahead and substitute the expression $r_2 + 2$ for r_1 into the first equation.

$\frac{48}{r_1} = \frac{32}{r_2}$	Using our first equation, substitute $r_2 + 2$ for r_1
$\frac{48}{r_2 + 2} = \frac{32}{r_2}$	An equivalent equation

Now we can solve this equation because we only have one variable, r_2, in the equation. Since this looks like a proportion, we can solve it by cross-multiplying.

$\frac{48}{r_2+2}=\frac{32}{r_2}$	Using the equivalent equation, cross-multiply with the denominators
$r_2\cdot\cancel{(r_2+2)}\cdot\frac{48}{\cancel{r_2+2}}=\frac{32}{\cancel{r_2}}\cdot\cancel{r_2}\cdot(r_2+2)$	Simplify
$48r_2=32(r_2+2)$	Distribute
$48r_2=32r_2+64$	Subtract $32r_2$ from both sides
$16r_2=64$	Divide both sides by 16
$r_2=4$	Kate's rate is 4 bike per hour

We find that Kate's rate is 4 bikes per hour.

Now we can use that value for r_2 to find the value for r_1, which is Jenny's rate for repairing bikes. To do that, we're going to use the second equation because it has what we're looking for, the rate for Jenny already isolated.

$r_1=r_2+2$	Using the second equation, substitute 4 in for r_2
$r_1=4+2$	Add 4 and 2
$r_1=6$	Jenny's rate is 6 bikes per hour

Jenny can repair 6 bikes per hour, while Kate can repair 4 bikes per hour.

SUMMARY

The **work, rate, and time relationship** can be expressed as: amount of work completed equals the product of the rate and time. Work can be a measure of the amount of things completed and is related to the speed they are completed and the amount of time spent. When rates work together, the **combined rate** is the sum of the individual rates. When **solving a work, rate, and time problem with a system of equations**, first define the variables in the problem.

FORMULAS TO KNOW

Work, Rate, and Time

$$work = rate \cdot time$$