Writing a System of Linear Equations

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WHAT'S COVERED

In this lesson, you will learn how to determine the system of linear equations that represent a given situation. Specifically, this lesson will cover:

1. Developing a System of Equations from a Situation

In order to determine if a situation can be represented by a system of equations, we need to consider how the variables are defined. Simply put, a system of equations consists of at least two equations that contain the same variables and the same variable definitions. Developing a system of equations is very similar to writing equations based on situations, but we need to take an additional step to confirm that the equations can be considered simultaneously (or all at the same time).

EXAMPLE A chemist is mixing two solutions of different concentrations in the lab in order to prepare for an experiment. She needs 100 mL of a 40% solution so she mixes a 50% solution with a 30% solution.

We want to represent this situation with equations, and then determine if the equations represent a system. Let's first define variables for what we do not know. We know she is mixing solutions, but we are not sure how much of each solution she is mixing. Therefore, we will define the variable *x* as milliliters of the 50% solution, and the variable *y* as milliliters of the 30% solution

$$x = \text{mL of 50\% solution}$$
$$y = \text{ml of 30\% solution}$$

Now that we have defined variables, we can construct our equations. We know that in total, she will have 100 mL of her solution for the experiment. So we can write an equation to show that the amount of 50% solution and the amount of 30% solution will total 100 mL. Since mL is included in our definitions for *x* and *y*, we won't need to write that in our equation:

$$x + y = 100$$

We can also write an equation to represent the two solutions mixing together to form a 40% solution, by multiplying the quantity by its concentration:

$$0.5x + 0.3y = 0.4(x + y)$$

Note that we can simplify this equation since we know that the sum of x and y is 100:

$$0.5x + 0.3y = 0.4(100)$$
$$0.5x + 0.3y = 40$$

Now we have two equations that were derived from this situation:

$$0.5x + 0.3y = 40$$
$$x + y = 100$$

Is this a system of equations? To determine this, we need to look at how our variables are defined in each equation. In both equations, x has the same definition: milliliters of 50% solution to be mixed. Likewise, y has the same definition in each equation: milliliters of 30% solution to be mixed. Therefore, we can conclude that this is a system of equations.

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2. Developing a System of Equations involving Cost and Quantity

Oftentimes as consumers, we scope out a few different stores to find the best deals on items we purchase.

↱ EXAMPLE Suppose you are shopping around for top soil and wood chips for your backyard gardening project. You compared prices at two different home improvement stores for bags of top soil and wood chips to put in your backyard garden.

- At Store A, top soil cost $1.30 per bag, and wood chips cost $8.50 per bag.
- At Store B, top soil costs $1.70 per bag, and wood chips cost $8.00 per bag.
- The total at Store A came to $83.60, while the total at store B came to $84.40.

We can write a system of linear equations to represent this situation, and eventually solve for the number of bags of top soil and wood chips we were looking to buy at each store. For now, we are just going to focus on developing the equations for this situation.

We are going to have two equations in our system: an equation for Store A, and an equation for Store B. In each equation, we are going to express the total cost as:

$$(\text{price of top soil bags})(\text{number of top soil bags}) + (\text{price of wood chip bags})(\text{number of wood chip bags}) = \text{total cost}$$

We know the prices of top soil and wood chips at each location, but we do not know the number of bags of top soil and the number of bags of wood chips. Since these are our unknowns, we'll use variables:

$$x = \text{number of top soil bags}$$
$$y = \text{number of wood chip bags}$$

So for Store A, our equation becomes:

$$1.30x + 8.50y = 83.60$$

This tells us that x number of top soil bags for $1.30 each plus y number of wood chip bags at $8.50 each comes to $83.60, which matches our situation.

For Store B, our equation becomes:

$$1.70x + 8.00y = 84.40$$

This tells us that x number of top soil bags for $1.70 each plus y number of wood chip bags at $8.00 each comes to $84.40, which matches our situation.

Take note of how the variables are defined in each equation. Because in each equation, x represents the number of top soil bags, and y represents the number of wood chip bags, these two equations represent a system and can be considered at the same time.

Now we have two equations that were derived from this situation:

$$\begin{aligned} 1.30x + 8.50y &= 83.60 \\ 1.70x + 8.00y &= 84.40 \end{aligned}$$

SUMMARY

When **developing a system of equations from a scenario**, we need to ensure that the variables in each equation have the same definition. The equations will be considered at the same time, meaning they will be solved together to determine the variables. One common scenario is **developing a system of equations involving cost & quantity**.